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ADAPTIVE PROCESSING FOR LANDSAT DATA

ROBERT B. CRANE AND JAMES F. REYER
Infrared and Optics Division

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ENVIRONMENTAL
RESEARCH INSTITUTE OF MICHIGAN
FORMERLY WILLOW RUN LABORATORIES, THE UNIVERSITY OF MICHIGAN
BOX 618 • ANN ARBOR • MICHIGAN 48107

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16. Abstract This report contains analytical and test results on the use of adaptive processing on LANDSAT data. The Kalman filter is used as a framework to contain different adapting techniques. An earlier report showed that all of the modifications that were made to the Kalman filter performed the functions for which they were designed. The previous tests were limited to aircraft MSS data. In this report, the same conclusions are reached when LANDSAT MSS data are used. It was found that adaptive processing could provide compensation for incorrect signature means, within limits. However, if the data were such that poor classification accuracy would be obtained when the correct means were used, then adaptive processing would not improve the accuracy and might well lower it even further.					
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PREFACE

This report describes part of a comprehensive and continuing program of research concerned with advancing the state-of-the-art in remote sensing of the environment from aircraft and satellites. The research is being carried out for the NASA Lyndon B. Johnson Space Center, Houston, Texas, by the Environmental Research Institute of Michigan (ERIM), formerly the Willow Run Laboratories of The University of Michigan. The basic objective of this multidisciplinary program is to develop remote sensing as a practical tool to provide the planner and decision-maker with extensive information quickly and economically.

Timely information obtained by remote sensing can be important to such people as the farmer, the city planner, the conservationist, and others concerned with problems such as crop yield and disease, urban land studies and development, water pollution, and forest management. The scope of our program includes: (1) extending the understanding of basic processes; (2) discovering new applications, developing advanced remote-sensing systems, and improving automatic data processing to extract information in a useful form; and also (3) assisting in data collection, processing, analysis, and ground-truth verification.

The research described here was performed under NASA Contract NAS9-14123, Task VII, and covers the period from 15 May 1974 through 14 March 1975, Dr. Andrew Potter has been Technical Monitor. The program was directed by R. R. Legault, Vice-President of ERIM, by J.D. Erickson, Project Director and Head of the Information Systems and Analysis Department, and by R.F. Nalepka, Principal Investigator and Head of the Multispectral Analysis Section. The ERIM number for this report is 109600-14-F.

The authors wish to acknowledge the direction provided by Mr. R.R. Legault, Dr. J.D. Erickson, and Mr. R.F. Nalepka. Many constructive discussions were held with H. Horwitz, R.J. Kauth, W. Richardson, and many others at ERIM.

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1

SUMMARY

This report covers the continuation of a study of the use of a Kalman filter for adaptive processing. Included are analytical and test results pertaining to LANDSAT data. An earlier report [1] describes the first portion of the study, including test results using aircraft MSS data.

The purpose of adaptive processing is to continuously update the mean vectors of the class signatures using the data itself to provide the updating thereby allowing a local signature to have more universal applicability for classification. Originally, the Kalman filter was chosen because it provided an ordered structure into which many attractive ad-hoc updating techniques could be fitted and better understood, with others perhaps being derived. A limitation usually found in Kalman filters, the requirement for excessive processing time and computer memory when a large number of states must be updated, was circumvented by the use of a simplified form of the filter. (Section 3.2 contains an equivalence relationship between the normal and simplified forms.)

In one form of the Kalman filter, the state variations are described as a Markov process. Occasionally, this description is replaced with an assumption of correlated variations. Because states are identified with the mean vectors of the class signatures, the correlated variation assumption was used, with a correlation length corresponding approximately to agricultural field sizes.

In our earlier report, we presented test results on the Kalman filter form of adaptive processing that were obtained by using aircraft MSS data. In that report, we showed that the modifications to the basic Kalman filter appropriate to decision-directed classification would perform the various optional functions for which they were designed such as line-by-line updating rather than pixel-by-pixel. In this report, it is shown that the same conclusion can be reached from the LANDSAT data test results.

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- [1] Crane, R.B., Adaptive Processing with a Decision-Directed Kalman Filter and Feature Extraction of Multispectral Data, Report No. 190100-31-T, Environmental Research Institute of Michigan, Ann Arbor, July 1974.

In one test it was found that the signature means did not have to be the means of the data classes. With adaptive processing the means can vary approximately 20% without degrading classification accuracy below that obtained with the measured means and non-adaptive processing. This result indicates that adaptive processing may be used to accomplish limited signature extension (i.e., to permit signature established under one set of measurement conditions to be successfully applied under slightly different condition). For situations where mean values differ by more than 20% and other signature extension techniques could transform the means so that their values were within the 20% range, adaptive processing could be used to find even better values. The same test showed that if the means were outside of a 35% range of values, then non-adaptive processing outperformed adaptive processing. This result is not particularly significant because unusually poor classification accuracy was obtained in either case. In fact, several test results are available that show that adaptive processing cannot improve classification accuracy when the non-adaptive classification accuracy is poor. An explanation may be that the Kalman filter is decision directed, requiring that most of the data points be classified accurately so that the points can be directed to update the correct means.

2

BACKGROUND AND INTRODUCTION

Multispectral scanner data acquired by remote sensing will usually cover large areas on the ground. The large amounts of data collected make machine processing extremely desirable for extracting information. To use machine processing, one must rely on some commonality within the data, usually spectral invariance among ground covers of the same class. The quality of the extracted information is limited by the extent to which such commonality exists, as well as by limited knowledge of the commonality.

One method of processing multispectral scanner data to extract information is to select data subsets, called training sets, for each of the classes, and use these to derive a signature for that class. The signature consists of a mean vector and a covariance matrix describing the data vectors from the training sets. Then, assuming that the signatures for each class accurately represent that class, a decision rule is defined which assigns each of the scanner data points to one of the classes (or to a null class).

There are many reasons why a class signature may not accurately represent that particular class at all times and places; 1) there may be an insufficient number of data points in the training set; 2) the class may be composed of subclasses with different reflectances and the signature could describe only one subclass; 3) atmospheric conditions may not be constant between the training and test areas; and, 4) the illumination and viewing geometry (position of the sun and the sensor relative to the ground resolution element) could differ for training and non-training data. Signatures should be the best obtainable so that classification accuracy is maximized, thereby maximizing the information extracted from the data.

One way to increase the accuracy of the signatures is to increase the number of training sets. If the training sets are located throughout the scene, one can use different signatures for different portions of the scene. However, there is a limit to the number of training sets that can be used which is quickly reached, beyond which the cost of obtaining the requisite ground truth becomes prohibitive for operational systems.

A second way to increase signature accuracy is to preprocess the signatures or data so that variations are accounted for, removed, or reduced. This method can be extremely effective for certain types of variations, and can greatly reduce the number of training sets required and hence the cost. However, this method becomes less effective when the variations are different for different classes.

A third way to increase signature accuracy and thereby classification accuracy is to use adaptive processing to update the signatures as the data are being processed. The basic idea is simple. Suppose that a number of points are identified as belonging to a certain class, and that the average value of these points is larger than the mean vector of the signature for that class. Then the signature is changed by increasing the mean vector. The amount of increase depends on the number of points, the amount by which the average exceeds the mean vector, and a factor which will be called the updating rate. This basic adaptive processing method, along with some imaginative variations, was used for processing multispectral scanner data [3]. The success of the initial effort led to the use of a Kalman filter.

A Kalman filter is used to update the mean vectors of the signatures while the data are being classified. We combine the mean vectors for all of the classes into one large state vector, which is then updated. The Kalman filter method has the advantages of being an iterative technique ideally suited for use with a digital computer, and of providing a general formulation or organization that combines many techniques for improving recognition. There are, however, three potential disadvantages: 1) The Kalman filter normally requires large amounts of computer memory; 2) It requires large amounts of computation time, and 3) there is a possibility of "capture".

The reason for the large memory requirement is that the joint statistics of the state variables (the covariance of the state vectors) must

[3]Marshall, R.E., F.J. Kriegler, and W. Richardson, Adaptive Multispectral Recognition of Wheat, Tenth Symposium on Adaptive Processes, Miami Beach, December 1971.

be stored, as must the covariance of the individual observations of these state variables (i.e., the crop signature covariances). These can be thought of respectively as the statistics of the underlying process (state vector) and the statistics of the noise of observation of that process.

The reason that the Kalman filter normally requires a large amount of computation time is that both of the above forms of covariance enter the calculation. The calculation time for each update varies as the product of state vector calculation time and noise calculation time. If the basic signal is n channels and there are m materials, then the state vector is mn in length, the state covariance is $nm \times nm$, and the noise covariance is $n \times n$ for each signature.

The statistics of the state vector were not known initially; these could only be learned by observation of the Kalman filter process itself. Hence an initial assumption had to be made: that the state covariance was a higher dimensional image of the average noise covariance. This assumption turned out to be fortuitous since it simplified the Kalman filter equations greatly, and both calculation times and memory requirements became negligible.

The third possible disadvantage mentioned above was the possibility of "capture", wherein the mean signature for material A becomes adapted to the true mean of material B. In this case material B has captured the signature for material A and, as a result, material B systematically becomes classified as material A. Capture can occur when false classifications cause the mean vectors of one or more classes to be updated using the wrong data points.

Use of the Kalman filter necessitated three assumptions, the justification for which is that the resulting updating equations increase classification accuracy. One assumption is that the recognition decisions are correct. Because this assumption is questionable, we borrowed a technique from our initial study [4] to modify the amount of updating by a confidence factor which reflects the uncertainty in the correctness of the decisions. Another assumption is that all of the covariance matrices of

[4] Kriegler, F.J., R.E. Marshall, H.H. Horwitz, and M.F. Gordon, Adaptive Multispectral Recognition of Agricultural Crops, Eighth International Symposium on Remote Sensing of Environment, ERIM, Ann Arbor, October 1972.

the signatures are equal. This assumption is made to derive the Kalman filter, and is not used when the decision rule is computed. A third assumption that we also make, is that the covariance of any two mean vectors is proportional to the same common covariance matrix. We do not anticipate sufficient ground truth in most data sets so that more accurate statistics could be used. We also assume that the data are Gaussian, or equivalently, that we want the optimum (in the mean square sense) linear updating equations.

We have built several functions into our Kalman filter. In its simplest form, the filter can be used to update, after each decision, the mean vector of the class that was recognized. One of the functions is the ability to update all mean vectors, when we wish to include interaction i.e., non-zero covariance between pairs of mean vectors.. Additionally, we can update after every line or fraction of a line has been classified, a feature which reduces the time required for processing a data set compared to updating after every point. We also have incorporated the confidence factor mentioned previously.

Thus far the discussion has been limited to updating the signature mean vectors. We also have the capability of estimating and updating angular dependence of the data. For large data sets it may prove desirable to update the angular dependence, because some of the reasons for needing an updating algorithm; e.g., atmospheric changes, are also reasons why the angular dependence of the mean vectors would change.

Another feature of this program is the capability to use auxiliary training fields to improve the updating accuracy and lessen the probability of capture. When this feature is used, the updated value of the mean vectors depends not only on the original signatures and the data already processed, but also on signatures of training sets to be processed downstream. The use of auxiliary training fields should reduce the ground truth requirement, because signatures of the auxiliary training fields are used for data collected both before and after the training sets, with continuity between training sets provided by the Kalman filter.

The original adaptive processing [3] and our first implementation of the Kalman filter were used with the quadratic (maximum likelihood) decision rule. One step in improving the adaptive processing procedure was to change to the linear decision rule [5]. Previous comparisons of the two decision rules showed that when the same number of data channels were used, the classification accuracy obtained with the two rules were approximately the same for test data, and the linear rule required approximately one-third the processing time of the quadratic rule. The fact that the linear rule provided slightly higher classification accuracy was felt to be statistically insignificant. A recent comparison made as part of the CITARS program [6] confirms the approximate classification accuracy equivalence of the two decision rules.

A test program was initiated to show the usefulness of the Kalman filter approach to adaptive processing. Initial results, using aircraft MSS data, indicated that the approach was useful for processing data gathered under a variety of conditions [1].

This report summarizes our modifications and testing of the Kalman filter for LANDSAT data. Simplifications of the theory presented in our earlier report are contained in Section 3. Also included are extensions to the theory that are applicable to LANDSAT data. Results of testing the modified Kalman filter are presented in Section 4. Conclusions and recommendations are presented in Section 5 which includes a recommended test plan applicable to the LACIE program.

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- [3] Marshall, R.E., F. J. Kriegler, and W. Richardson, Adaptive Multispectral Recognition of Wheat, Tenth Symposium on Adaptive Processes, Miami Beach, December 1971.
 - [5] Crane, R.B. & W. Richardson, Performance Evaluation of Multispectral Scanner Classification Methods, Eighth International Symposium on Remote Sensing of Environment, Willow Run Laboratories of the Institute of Science and Technology, The University of Michigan, Ann Arbor, October 1972.
 - [6] Malila, William A., Daniel P. Rice and Richard C. Cicone, "Final Report on the CITARS Effort by the Environmental Research Institute of Michigan" ERIM Report No. 109600-12-F, Environmental Research Institute of Michigan, Ann Arbor, Michigan, February 1975.
 - [1] Crane, R.B., Adaptive Processing with a Decision-Directed Kalman Filter and Feature Extraction of Multispectral Data, Report No. 190100-31-T, Environmental Research Institute of Michigan, July 1974.

DESCRIPTION OF MODIFIED KALMAN FILTER

In previous work [1] a general formulation of the Kalman filter was presented and the way such a filter could be used to update the signature mean vectors was discussed in qualitative terms. Various terms of the equations were identified with observed phenomena. The dependence of the equations upon the statistical properties of the data was shown explicitly. One possible approximation to the statistics was shown to lead to a simplification of the equations along with greatly reduced computational and memory requirements of a general purpose digital computer. Also shown were some extensions of the filter that appeared at the time to be most useful for either aircraft or satellite data.

In this section we repeat the analytical description of the Kalman filter, although the approach is different. We then show a general equivalence between the Kalman filter and a simplified filter. Finally, the equivalence relationship is used to show various modifications to the Kalman filter that have proven useful.

3.1 BASIC KALMAN FILTER

Beginning with one version of the Kalman filter equations, the first problem is to estimate X_k using measurements Z_0, \dots, Z_k . We identify X_k with a vector composed of all of the class mean vectors, and identify the Z_i with data vectors. An optimum estimate of X_k , which we shall label \hat{X}_k , must be found which minimizes:

$$M = E\{[X_k - g(Z)]^t [X_k - g(Z)]\}. \quad (1)$$

Here, $g(Z)$ represents a function of Z_0, \dots, Z_k . The probability distribution of X_k and Z can be written as

$$f(X_k, Z) = f(X_k | Z) f(Z) \quad (2)$$

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- [1] Crane, R.B., Adaptive Processing with a Decision-Directed Kalman Filter and Feature Extraction of Multispectral Data, Report No. 190100-31-T, Environmental Research Institute of Michigan, Ann Arbor, July 1974.

so Equation 1 becomes

$$M = \int f(Z) \left[\int [X_k - g(Z)]^t [X_k - g(Z)] f(X_k|Z) dX_k \right] dZ \quad (3)$$

Now M is nonnegative, so we minimize the bracketed integral for every Z . In this integral, $g(Z)$ is a constant, which is minimized if $g(Z)$ is the mean of X_k . Thus the minimum mean square estimate of X_k is

$$\hat{X}_k = g(Z) = E(X_k|Z) \quad (4)$$

Equation 4 can be rewritten when X_k and Z are jointly gaussian in the form

$$E(X_k|Z) = E(X_k Z^t) [E(Z Z^t)]^{-1} Z \quad (5)$$

Equation 5 is true for any pair of jointly gaussian vectors, and will be used repeatedly in the development. We now define an error vector

$$\tilde{X}_k = \hat{X}_k - X_k \quad (6)$$

and note that the minimization operation served to minimize the norm of \tilde{X}_k i.e., see Equation 1. Using Equation 5, it can be shown that:

$$E(\tilde{X}_k \hat{X}_k^t) = 0 \quad (7)$$

$$E(\tilde{X}_k Z^t) = 0 \quad (8)$$

The Kalman filter is now developed using the state model just described. It is assumed that one can update after every line of data, rather than after every data point. Each data point is related to the state by the equation

$$Z_{ki} = H_{ki} X_k + v_{ki} \quad (9)$$

where H_{ki} is a matrix that picks the mean vector of the class, after classifications, from X_k . The noise vector, v_{ki} , is measurement noise, with statistics

$$E(v_{ki}) = 0 \quad (10)$$

$$E(v_{ki} v_{jl}^t) = \delta_{kj} \delta_{il} R \quad (11)$$

One measurement vector will be used for each line, and is given by

$$Z_k = \sum_{i=1}^N C_{ki} z_{ki} \quad (12)$$

The C_{ki} are weighting factors which represent the confidence that the classification decision for each data point was correct,

Combining Equations 9 and 12 gives

$$Z_k = H_k X_k + v_k \quad (13)$$

where

$$H_k = \sum_{i=1}^N C_{ki} H_{ki} \quad (14)$$

$$v_k = \sum_{i=1}^N C_{ki} v_{ki} \quad (15)$$

and v_k is a measurement noise vector for the k^{th} line. To find the statistics, equations (1) and (11) are used:

$$\begin{aligned} E(v_k) &= 0 \\ E(v_k v_j^t) &= \delta_{kj} \sum_{i=1}^N C_{ki}^2 R \end{aligned} \quad (16)$$

We have now completed the preliminary derivation of the filter problem. From Equation 4 it is seen that one must find

$$\begin{aligned} X_k &= E(X_k | Z_0, \dots, Z_k) \\ &= E(X_k | Z_0, \dots, \tilde{Z}_k) \end{aligned} \quad (17)$$

where

$$\begin{aligned} \tilde{Z}_k &= Z_k - H_k \Phi \hat{X}_{k-1} \\ &= H_k (W_{k-1} - \Phi \hat{X}_{k-1}) + v_k \end{aligned} \quad (18)$$

The second equality in Equation 17 occurs because \hat{X}_{k-1}^v is a linear combination of Z_0, \dots, Z_{k-1} . Equation 8 can be used to show that:

$$E(Z_j \hat{Z}_k^v) = 0 \quad (19)$$

for $j < k$. Using Equation 5 it is seen that Equation 17 becomes

$$\begin{aligned} E(X_k | Z_0, \dots, \hat{Z}_k^v) &= E(X_k | Z_0, \dots, Z_{k-1}) + E(X_k | \hat{Z}_k^v) \\ &= \phi \hat{X}_{k-1} + E(X_k | \hat{Z}_k^v) \end{aligned} \quad (20)$$

We now use Equation 5 to find

$$E(X_k | \hat{Z}_k^v) = E(X_k \hat{Z}_k^v) [E(\hat{Z}_k \hat{Z}_k^v)]^{-1} \hat{Z}_k^v \quad (22)$$

Evaluating the separate parts of Equation 22,

$$E(X_k \hat{Z}_k^v) = P_k' H_k^t \quad (22)$$

where

$$P_k' = \phi P_{k-1} \phi^t + Q_{k-1} \quad (23)$$

$$E(w_j \hat{w}_k^t) = \delta_{jk} Q_k \quad (24)$$

$$P_k = E(\hat{X}_k \hat{X}_k^t) \quad (25)$$

and

$$E(\hat{Z}_k \hat{Z}_k^t) = H_k P_k' H_k^t + R \quad (26)$$

Combining Equations 20-26, we rewrite Equation 20 as

$$\hat{X}_k = \phi \hat{X}_{k-1} + K_k \hat{Z}_k^v \quad (27)$$

where

$$K_k = P_k' H_k^t (H_k P_k' H_k^t + R)^{-1} \quad (28)$$

$$\hat{X}_k = (I - K_k H_k) (\Phi \hat{X}_{k-1} - W_{k-1}) + K_k v_k \quad (29)$$

To complete the development, we evaluate Equation 25;

$$P_k = P_k' - K_k H_k P_k' \quad (30)$$

The adaptive processing equations that could be used are Equations 18, 23, 27, 28, and 30.

3.2 EQUIVALENCE OF SIMPLIFIED KALMAN FILTER

The main purpose of this section is to explain why previous adaptive processing developments can be put into a simplified form. An additional purpose is to provide logical justification **for**, and description of, a method whereby new developments can be formulated directly in the simplified form. The advantages of this simplified form over the normal form of adaptive processing are reduced computation time and storage requirements.

We start by writing the equations that define the Kalman filter that we have been using.

$$X_k = X_{k-1} + \omega_{k-1} \quad (31)$$

$$E(\omega_k \omega_i^t) = Q_k \delta(k, i) \quad (32)$$

$$Z_k = H_k X_k + n_k \quad (33)$$

$$E(n_k n_i^t) = b_k R \delta(k, i) \quad (34)$$

In these equations, as before, X_k is a state vector composed of the mean vectors of all of the classes that exist at a sampling time t_k . The vector ω_k is a random vector, normally distributed, with mean zero and covariance defined by Eq. 32. Equation 33 defines the measurement vector Z_k , identified to be the MSS data vector, as a function of 1) the state vector, 2) a pointing matrix, H_k and 3) a random Gaussian vector, n_k . The

pointing matrix uses recognition results to select the components of the state vector that form the mean vector of the class that was recognized. The random vector, n_k , has zero mean and covariance defined by Eq. 34.

Because all random processes have a normal distribution, the minimum mean-squared estimator for X_k is

$$\hat{X}_k = E(X_k | Z_0, \dots, Z_j) \quad (35)$$

The matrix $T_{k,i}$ is defined to be

$$T_{ki} = E(X_k X_i^t) \quad (36)$$

Note that by using Eqs. 31 and 32, one can show that

$$T_{k,k} = T_{k-1,k-1} + Q_{k-1} \quad (37)$$

We are now ready to evaluate Eq. 35. A vector Z is defined as

$$Z = \begin{pmatrix} Z_0 \\ \vdots \\ Z_j \end{pmatrix} \quad (38)$$

and Eq. 35 written as

$$\hat{X}_k = E(X_k | Z) = E(X_k Z^t) [E(Z Z^t)]^{-1} Z \quad (39)$$

Components of $E(X_k Z^t)$ and $E(Z Z^t)$ are

$$E(X_k Z_i^t) = E[X_k (H_i X_i + n_i)^t] = T_{ki} H_i \quad (40)$$

and

$$E(Z_i Z_\ell^t) = E[(H_i X_i + n_i)(H_\ell X_\ell + n_\ell)^t] = H_i T_{i\ell} H_\ell^t + b_i R \delta(i, \ell) \quad (41)$$

The formulation of the Kalman filter is now complete. This formulation is equivalent to the iterative formulation that we have been using. The next step is to introduce a second Kalman filter which will later be related to the simplified form.

$$\eta_k = \eta_{k-1} + Y_{k-1} \quad (31a)$$

$$E(Y_k Y_i^t) = \theta_k \delta_{k,i} \quad (32a)$$

$$\zeta_k = M_k^t \eta_k + \sigma_k \quad (33a)$$

$$E(\sigma_k \sigma_i) = b_k \delta(k,i) \quad (34a)$$

$$\hat{\eta}_k = E(\eta_k | \zeta_0, \dots, \zeta_j) \quad (35a)$$

$$E(\eta_k \eta_i^t) = S_{k,i} \quad (36a)$$

$$S_{k,k} = S_{k-1,k-1} + \theta_{k-1} \quad (37a)$$

$$\zeta = \begin{pmatrix} \zeta_0 \\ \vdots \\ \zeta_j \end{pmatrix} \quad (38a)$$

$$\hat{\eta}_k = E(\eta_k \zeta^t) [E(\zeta \zeta^t)]^{-1} \zeta. \quad (39a)$$

$$E(\eta_k \zeta_i^t) = S_{k,k} M_i \quad (40a)$$

$$E(\zeta_i \zeta_\ell^t) = M_i^t S_{i\ell} M_\ell + b_i \delta(i,\ell) \quad (41a)$$

To relate the two formulations; we rewrite Eq. 39a as

$$\eta_k = \alpha_k \zeta \quad (42)$$

where

$$\alpha_k = E(\eta_k \zeta^t) [E(\zeta \zeta^t)]^{-1} \quad (43)$$

Next, we make the assumptions

$$Q_k = \Theta_k \otimes R \quad (44)$$

$$H_k = M_k^t \otimes I \quad (45)$$

$$T_{oo} = S_{oo} \otimes R \quad (46)$$

where the symbol \otimes represents the Kronecker product. Using Eqs. 46, 31, and 32, one can show that

$$T_{ik} = S_{ik} \otimes R \quad (47)$$

Next, Eq. 41 is evaluated

$$\begin{aligned} H_i T_{il} H_l^t + b_i R \delta_{i,l} &= (M_i^t \otimes I) (S_{il} \otimes R) (M_l \otimes I) + b_i R \delta_{i,l} \quad (48) \\ &= (M_i^t S_{il} M_l + b_i \delta_{il}) \otimes R \end{aligned}$$

and, by using Eq. 41a, we see that

$$E(ZZ^t) = E(\zeta \zeta^t) \otimes R \quad (49)$$

In a similar manner,

$$T_{ki} H_i^t = (S_{ki} \otimes R) (M_i \otimes I) = S_{ki} M_i \otimes R \quad (50)$$

so that

$$E(X_k Z^t) = E(\eta_k \zeta^t) \otimes R \quad (51)$$

Combining Eqs. 39, 49, 51 and 53, results in

$$\hat{X}_k = (\alpha_k \otimes I) Z \quad (52)$$

Equation 52 is an important result. It says that if one can find the optimum filter for the simplified system, defined by Eqs. 31a to 41a, one has also found the simplified form of the filter for the system defined by Eqs. 31 to 41. In addition, one can also show the relationship between the covariance matrices for the state estimation errors for the two systems, which are

$$\rho_k = E[(\hat{\eta}_k - \eta_k)(\hat{\eta}_k - \eta_k)^t] \quad (53)$$

$$P_k = E[(\hat{X}_k - X_k)(\hat{X}_k - X_k)^t] = \rho_k \otimes R \quad (54)$$

3.3 AUXILIARY FIELDS

The use of auxiliary fields was introduced in [1]. Auxiliary fields provide a method of reducing the undesirable characteristic of signature capture in an updating method, including the Kalman filter. In multispectral scanner data processing, signature capture occurs when the mean of one class actually describes the data from another class. Data from one (or more) material is recognized incorrectly, clearly an undesirable situation. Of course, the same misclassification can occur without updating, and it is possible that updating may eliminate the problem.

Auxiliary fields are ground truth fields which are located throughout the scene. The mean values of the data from the fields are considered to be additional measurement vectors, and are used to determine how much the mean vectors are to be updated. Because the auxiliary fields are correctly identified, their use should tend to overcome capture caused by incorrect classification of other data vectors.

The formulation of the updating equations to include the auxiliary fields has already been presented. Equation 35 is interpreted so that the measurements $Z_j, j > k$ denote the measurements from the auxiliary fields. The updating equation is (39), which is shown in simplified form in [1].

3.4 COLORED NOISE

One of the assumptions used for the Kalman filter is that variations in the state vector between sampling times are uncorrelated. (This assumption is described in Eq. 24.) We now describe a method whereby this assumption can be removed. First the state vector is formed in the following manner.

$$\begin{aligned} X_k &= \begin{pmatrix} Y_k \\ \omega_k \end{pmatrix} = \begin{pmatrix} I & I \\ 0 & \alpha I \end{pmatrix} \begin{pmatrix} Y_{k-1} \\ \omega_{k-1} \end{pmatrix} + \begin{pmatrix} 0 \\ n_{k-1} \end{pmatrix} \\ &= \Phi X_{k-1} + W_{k-1} \end{aligned} \quad (55)$$

The Y_k is the vector composed of all of the class mean vectors. The variation between Y_{k-1} and Y_k is

$$Y_k = Y_{k-1} + \omega_{k-1} \quad (56)$$

where

$$\omega_k = \alpha \omega_{k-1} + n_{k-1} \quad (57)$$

-
- [1] Crane, R.B., Adaptive Processing with a Decision-Directed Kalman Filter and Feature Extraction of Multispectral Data, Report No. 190100-31-T, Environmental Research Institute of Michigan, Ann Arbor, July, 1974.

The basic difference between this development and that found in [1] can be explained by Equation 52. In the previous developments, the state variations for different sampling times were independent. In this development, they are dependent. To show the dependence, we start with

$$E(n_k n_j^t) = R_n \delta_{kj} \quad (58)$$

which is similar to the assumption in [1].

We find

$$E(\omega_k \omega_k^t) = \alpha^{2i} E(\omega_{k-i} \omega_{k-i}^t) + \frac{1-\alpha^{2i}}{1-\alpha^2} R_n \quad (59)$$

for all $i \leq k$. For large k and $|\alpha| < 1$,

$$E(\omega_k \omega_k^t) \approx \frac{1}{1-\alpha^2} R_n \quad (60)$$

The correlation of the ω_k is

$$E(\omega_k \omega_j^t) = \alpha^{k-j} E(\omega_{k-j} \omega_{k-j}^t) \quad (61)$$

for $k > j$. If

$$\alpha = e^{-\frac{1}{n_0}} \quad (62)$$

then for large k ,

$$E(\omega_k \omega_j^t) \approx e^{-\frac{(k-j)}{n_0}} \left[\frac{1}{1-\alpha^2} R_n \right] \quad (63)$$

Thus n_0 can be considered to be analogous to a correlation length.

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- [1] Crane, R.B., Adaptive Processing with a Decision-Directed Kalman Filter and Feature Extraction of Multispectral Data, Report No. 190100-31-T, Environmental Research Institute of Michigan, Ann Arbor, July 1974.

The simplified equations are now found by making the following assumptions:

$$\Phi = \begin{pmatrix} 1 & 1 \\ 0 & \alpha \end{pmatrix} \otimes I \quad (64)$$

$$Q = \begin{pmatrix} 0 & 0 \\ 0 & R_n \end{pmatrix} \quad (65)$$

$$R_n = \theta \otimes R \quad (66)$$

$$P_k = \begin{pmatrix} a_k & b_k \\ t & d_k \end{pmatrix} \otimes R \quad (67)$$

$$H_{ki} = (M_{ki}^t \ 0) \otimes I \quad (68)$$

$$M_k = \sum_{i=1}^N C_{ki} M_{ki} \quad (69)$$

The Kronecker product, \otimes , is described and used in [1]. The vector M_{ki} represents the classification decision for the data point, and is composed of ones and zeros. The simplification of the filter equations is straightforward and tedious, so will be omitted. The updating equations become

$$\hat{Y}_k = \hat{Y}_{k-1} + \hat{\omega}_{k-1} + (\Phi_{k1} \otimes I) \hat{Z}_k \quad (70)$$

$$\hat{\omega}_k = \alpha \hat{\omega}_{k-1} + (\Phi_{k2} \otimes I) \hat{Z}_k \quad (71)$$

where

$$\hat{Z}_k = Z_k - (M_k^t \otimes I) \hat{Y}_{k-1} - (M_k^t \otimes I) \hat{\omega}_{k-1} \quad (72)$$

[1] Crane, R.B., Adaptive Processing with a Decision-Directed Kalman Filter and Feature Extraction of Multispectral Data, Report No. 190100-31-T, Environmental Research Institute of Michigan, Ann Arbor, July 1974.

ϕ_{k1} and ϕ_{k2} are defined by

$$\phi_{k1} = \frac{a_k^t M_k}{D} \quad (73)$$

$$\phi_{k2} = \frac{b_k^t M_k}{D} \quad (74)$$

with recursive relations:

$$b_{k+1} = a_k + b_k + b_k^t + d_k - \frac{(a_k + b_k)^t M_k M_k^t (a_k + b_k)}{D} \quad (75)$$

$$b_{k+1} = \alpha \left[b_k + d_k - \frac{(a_k + b_k)^t M_k M_k^t b_k}{D} \right] \quad (76)$$

$$d_{k+1} = \alpha^2 \left[d_k - \frac{b_k^t M_k M_k^t b_k}{D} \right] + \theta \quad (77)$$

where

$$D = M_k^t a_k M_k + \sum_{i=1}^N C_{ki}^2 \quad (78)$$

Some obvious characteristics of this technique should be noted. The size of the state vector is increased; with m materials and n channels, the state vector becomes $2mn$ rather than mn . The error matrix that must be stored becomes 4 times as large. The increased dimensionalities may not be too important, because the storage requirement for the filter is not large. Also, we do not expect a large increase in processing time. The filter without the correlated state variations increased processing

time, as compared to non-adaptive processing, by either 3% or 11%, the smaller percentage increase occurring when the quadratic decision rule was used. Most of the computation time was used on operations required for each data point, e.g., forming $C_{ki}Z_{ki}$, rather than for the updating. With the new method, only the updating equations have been changed.

4

EMPIRICAL EVALUATION OF MODIFIED KALMAN FILTER

A test program was initiated to show the usefulness of the Kalman filter approach to adaptive processing of LANDSAT data. Selected data sets were chosen which contained agricultural data since data could not be tested for every conceivable application and under every possible atmospheric condition. Indications of the usefulness of the technique for other applications can be inferred from the results presented in this section. However, additional testing should be performed so that parameters can be chosen to ensure that maximum benefits can be realized.

4.1 EFFECT OF DECISION RULE ON CLASSIFICATION ACCURACY

The major purpose of the adaptive processing now being developed is to correct the decision rule used for classification of multispectral scanner data. We now adapt the mean signature vectors which, with the covariance matrices, are used to determine the decision rule. An additional function sometimes employed is to update preprocessing transformations by updating an estimation of the angular variation of the data. Insight into the usefulness of the updating can be gained by determining how important it is to have the correct decision rule. We have determined the error rate for different decision rules applied to normal data from two classes with the same covariance matrix and the same a priori probabilities.

When the covariance matrices are equal no generality is lost in assuming unit distance between the means because the covariance matrix of either class is σ^2 times the identity matrix. A linear decision rule is assumed which can be characterized by two numbers, the distance from the origin, q , and the complement, ϕ , of the angle that the decision plane crosses the line joining the two means. See Figure 1. The optimum values are $q = 1/2$ and $\phi = 0^\circ$.

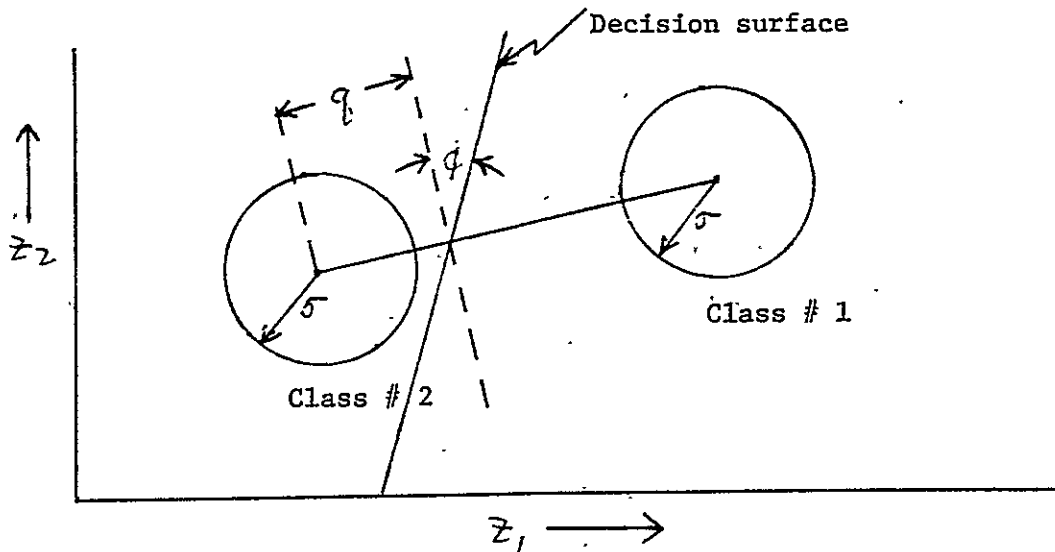


FIGURE 1. GEOMETRY OF TWO-CLASS DECISION RULE

For different values of q , ϕ , and σ , we have computed an average error rate, which is the average of the two types of error (choosing the second class, given data from the first class and vice versa). Figure 2 shows the effect of the choice of q for different values of $\sigma/\cos\phi$. The value of q is most important when $\sigma/\cos\phi$ is small and q is not near 0.5. Otherwise, small changes in q have a very small effect on the error rate. The same equations were plotted in Figure 3 with $\phi = 45^\circ$. With such a large error in ϕ , it is not surprising that the choice of q is not very important unless σ is rather small.

The effect of ϕ is shown in Figures 4 and 5 for two different values of q . Once again, small changes in ϕ have very small effect on the error rate. When ϕ is large, there can be a large error rate.

Recently, there has been an interest in estimating total acreage of wheat from a data set. In our simple example, we can consider the probability, P , of deciding the second class. This probability is shown in Figure 6, for different values of q and $\sigma/\cos\phi$. If $\sigma/\cos\phi$ is very small,

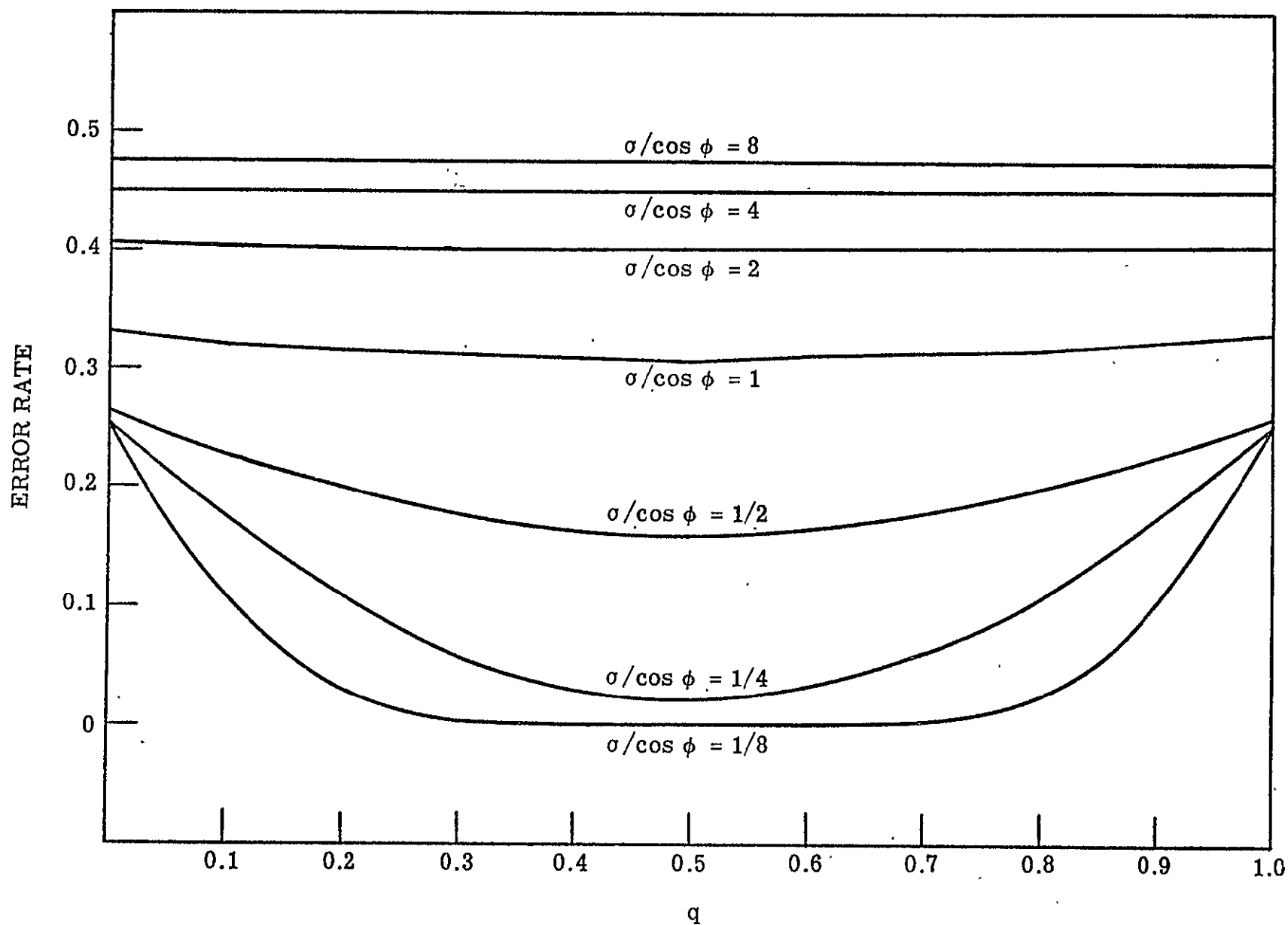


FIGURE 2: EFFECT OF THE POSITION OF THE DECISION SURFACE ON CLASSIFICATION ERROR.

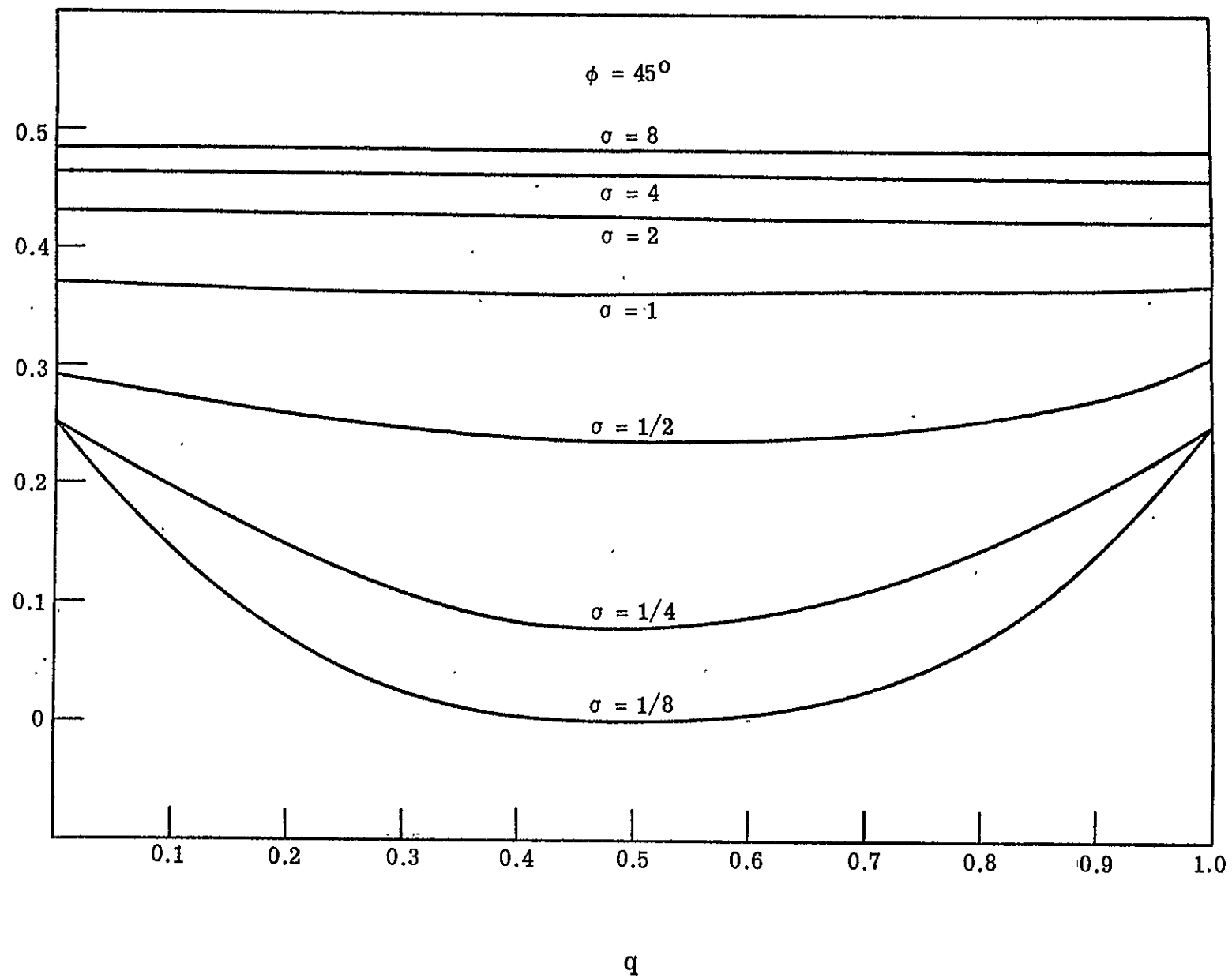


FIGURE 3: EFFECT OF DISPLACEMENT OF THE DECISION SURFACE ON CLASSIFICATION ERROR WHEN THE SURFACE IS ROTATED 45° FROM OPTIMUM.

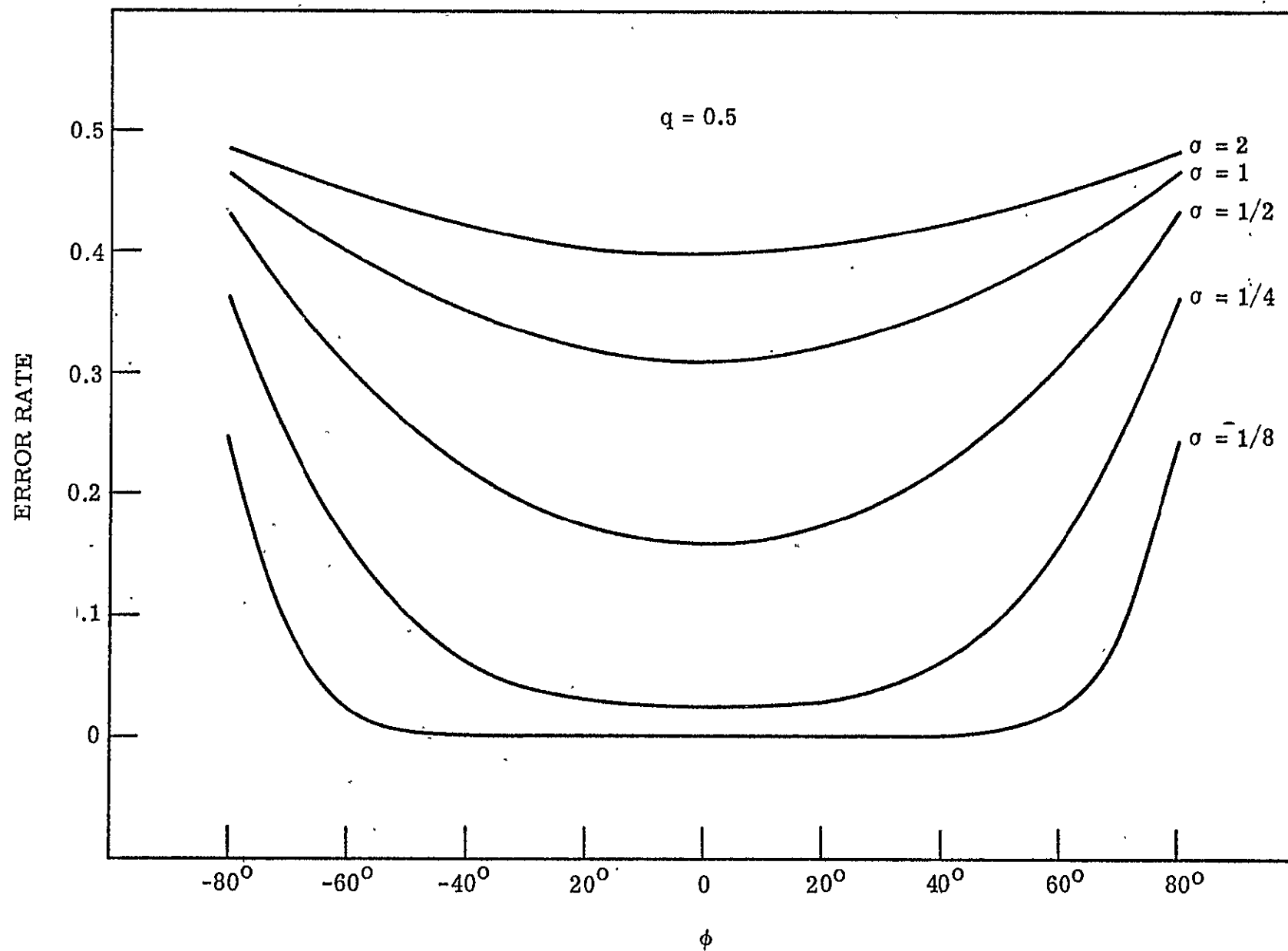


FIGURE 4: EFFECT OF ROTATION WITHOUT DISPLACEMENT OF THE DECISION SURFACE ON CLASSIFICATION ERROR.

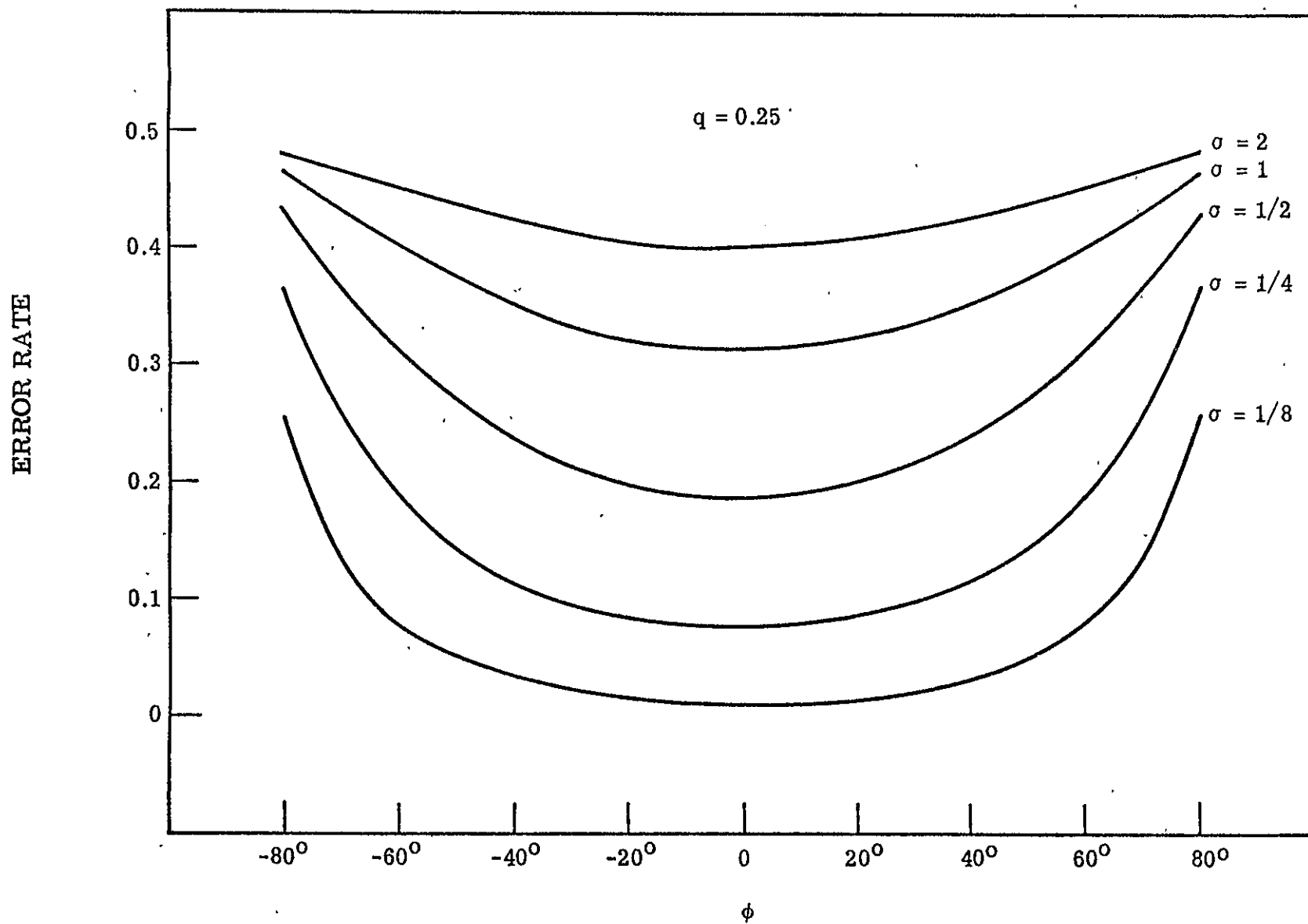


FIGURE 5: EFFECT OF ROTATION IN THE PRESENCE OF THE DECISION SURFACE ON CLASSIFICATION ERROR.

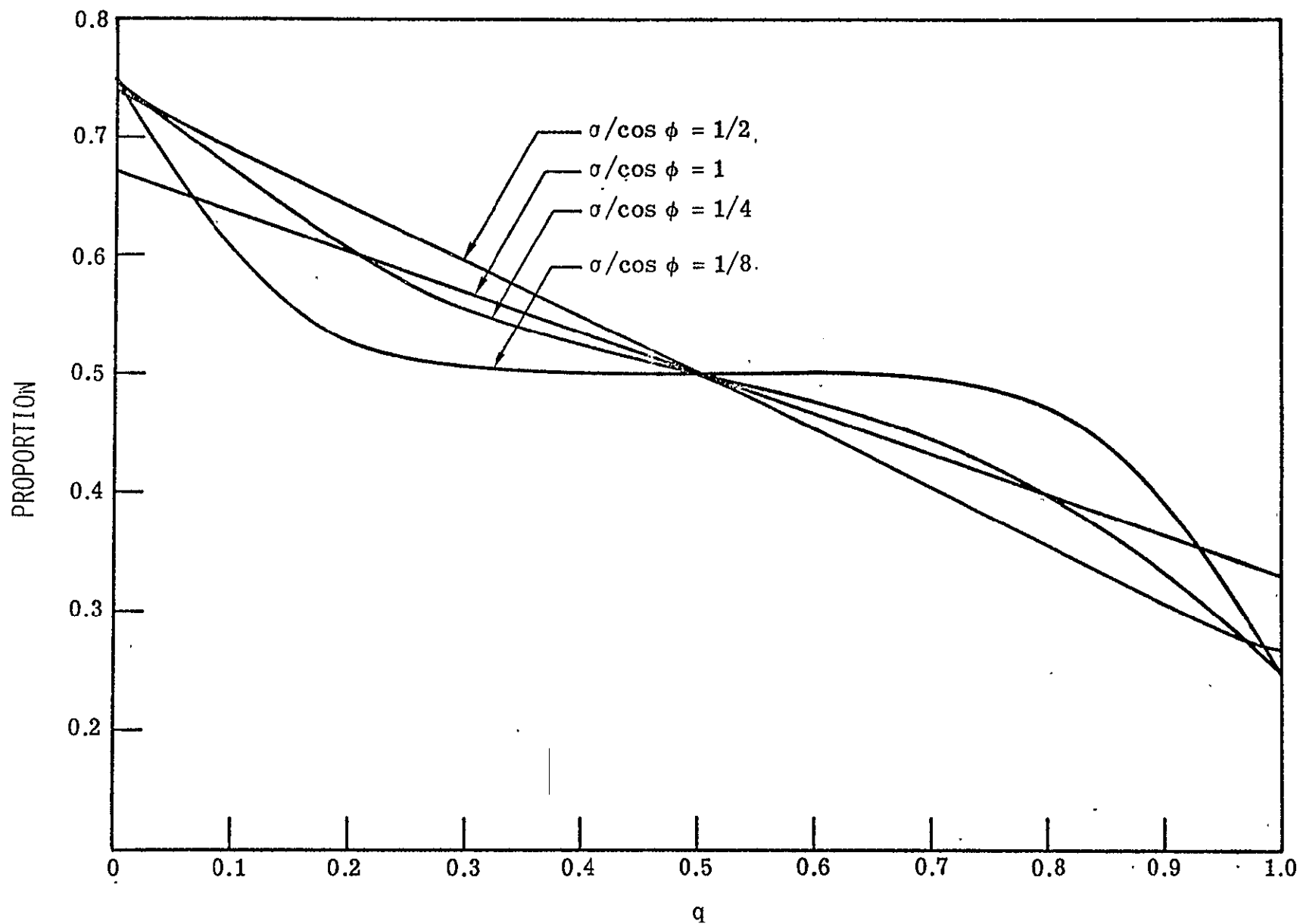


FIGURE 6: EFFECT OF DISPLACEMENT OF THE DECISION SURFACE ON THE PROPORTION OF DATA POINTS IDENTIFIED AS ONE OF THE CLASSES.

say $\leq 1/8$, then there is a range of q for which the probability of choosing the second class is approximately the correct value of 0.5. If $\sigma/\cos\phi$ is $\leq 1/2$, then there is an approximately linear relationship between p and q :

$$P \approx 1/2 + (1/2 - q)[1/2 - \phi(\sigma/\cos\phi)] \quad (79)$$

where

$$\phi(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \quad (80)$$

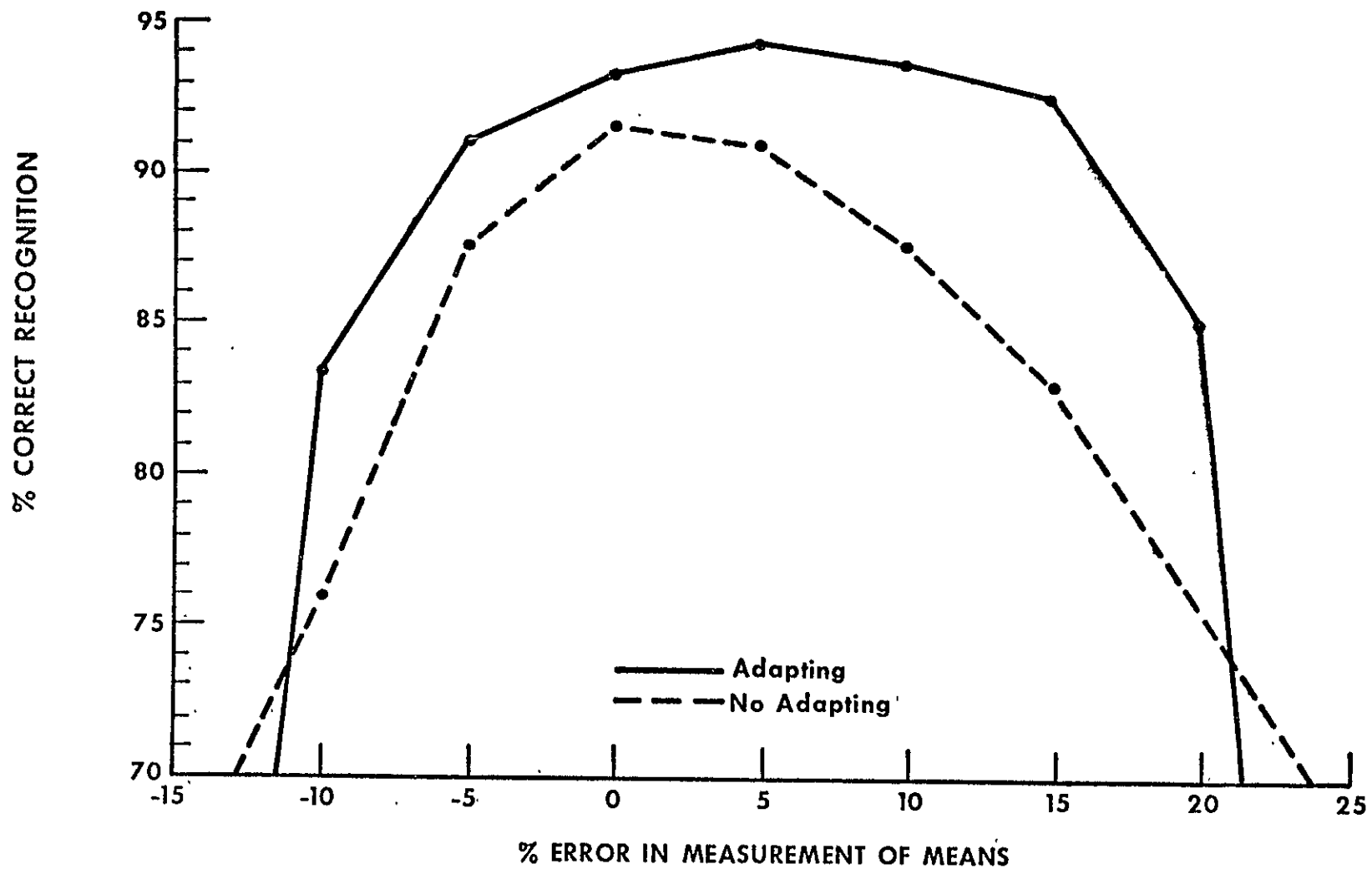
Let us now consider the usefulness of adaptive processing. When the criterion is minimum error rate, it appears that adaptive processing will be most useful when there are slowly varying changes in the means and the covariance matrices are not large compared to the separation of the means. For small deviations in estimating the means the error rate will be small so the adapting can correct the deviations. Thus if the means change slowly, the adaptive processing can track the means. If the means are not changing, the adaptive processing will find the correct means, but there will be little effect on the error rate. For large deviations, the error rate will be greater, and either the adaptive processing will respond slowly, but correctly, or capture will occur.

When the criterion of excellence is total acreage, a different picture emerges, at least for LANDSAT data. The linear relationship will hold for q , so that any percentage of recognitions can be obtained. If q is incorrect, then the farther ϕ is from the correct value the better the acreage estimation. It appears that average error rate is a better criterion for evaluating the usefulness of adaptive processing, even though small changes in the decision rule parameter may not be noticeable.

4.2 PROCESSING WITH INCORRECT MEANS

One of the advantages of adaptive processing is that it may enable the use of signatures that are somewhat in error. To test this concept, we tried different signatures, with and without adapting, on LANDSAT data. The fields were divided so that one half of the fields were test fields and the remaining fields were training fields. Both sets of fields were distributed through the scanned area. For each test result we increased or decreased all signature mean vectors by the same percentage and computed the percent correct recognition. The results are shown in Figure 7, which shows the percent correct recognition as a function of percent change in the means for both adaptive and non-adaptive processing. The means can vary approximately 20% with adaptive processing without degrading classification accuracy below that obtained with the measured means and non-adaptive processing. For 90% processing accuracy, the means can vary either 9% or 23%, depending upon whether non-adaptive or adaptive processing is used. For 85% accuracy, these numbers change to 19% and 29%. When we used very large percentage changes, neither method produced acceptable accuracy, although the non-adaptive processing would be preferred because capture cannot occur.

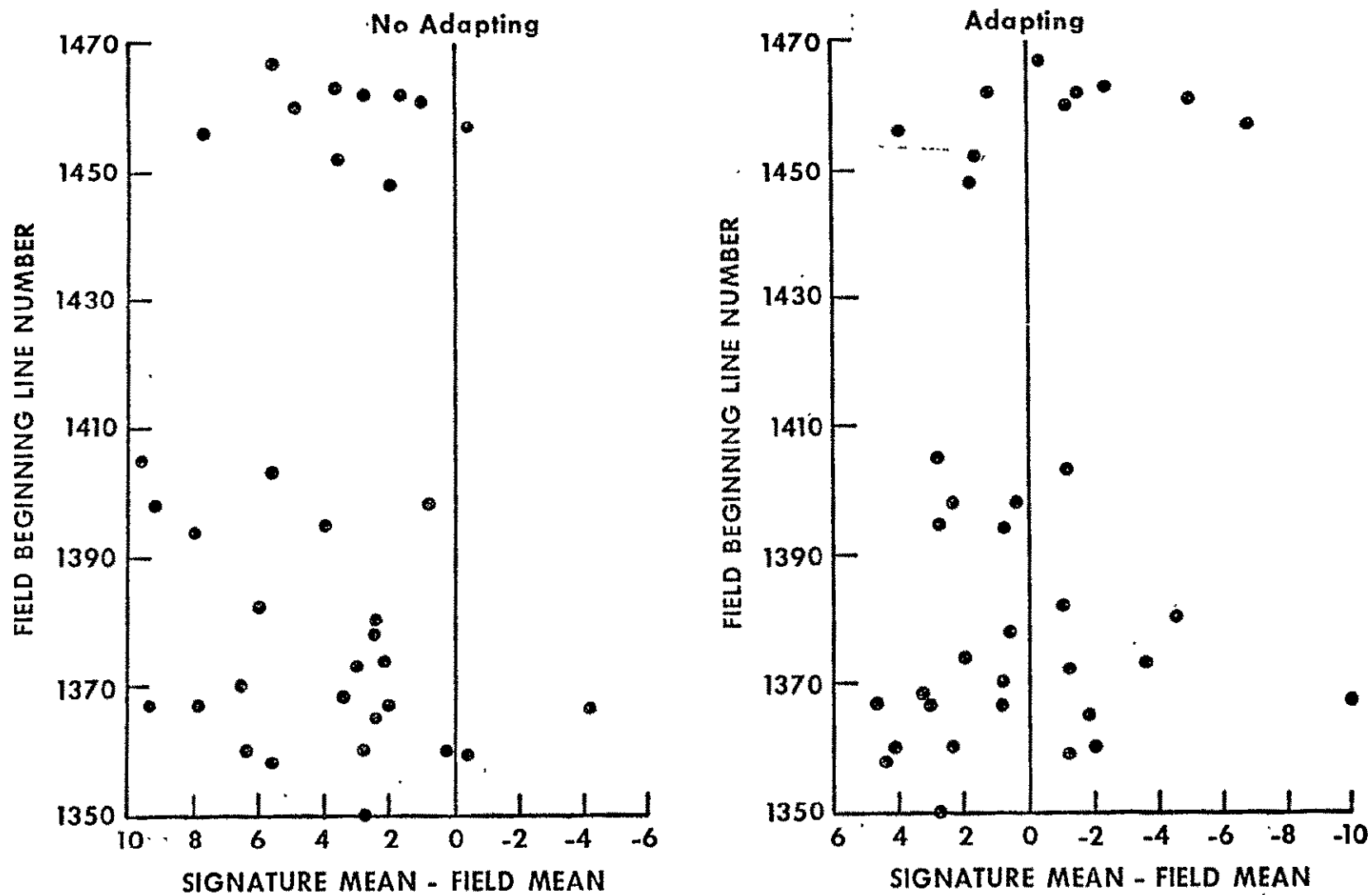
A visualization of the effects of adaptive processing can be seen in Figure 8, obtained from the same data and signatures as were used for Figure 7. The means of the ground-truthed fields were computed and compared, for each field, with the signature mean. Of course, without adapting, the signature mean is fixed while with adapting the mean is modified as the data changes. These differences are shown as a function of the beginning line number (indicative of the order of processing) for both non-adaptive and adaptive processing. For adaptive processing, the field means are centered around the true means, although there is a significant deviation which is caused by inter-field variations within each class. For non-adaptive processing, the field means are not centered around the signature means.



TEST FIELDS FROM N. DAKOTA ERTS DATA

FIGURE 7: CLASSIFICATION ACCURACY WITH DECISION RULES DERIVED FROM INCORRECT MEANS





N. DAKOTA ERTS DATA
+15% Error in all means



FIGURE 8: DIFFERENCES BETWEEN FIELD-CENTER MEANS AND DECISION-RULE MEANS WITH AND WITHOUT ADAPTING.

One can see the manner in which the signature means are updated in Figure 9 which shows the mean values in channel 2 (LANDSAT channel 5) for four classes. The values shown for line 1350 are the values obtained from the signatures. Note the gradual change in values as the line number increases. A slightly different set of curves is shown in Figure 10 which was obtained by changing the Kalman filter program so that three auxiliary fields would be used. The stars indicate the auxiliary fields, which are on the mean curves because the program is written so that the means coincide with the means of the auxiliary fields at the time the fields are being processed.

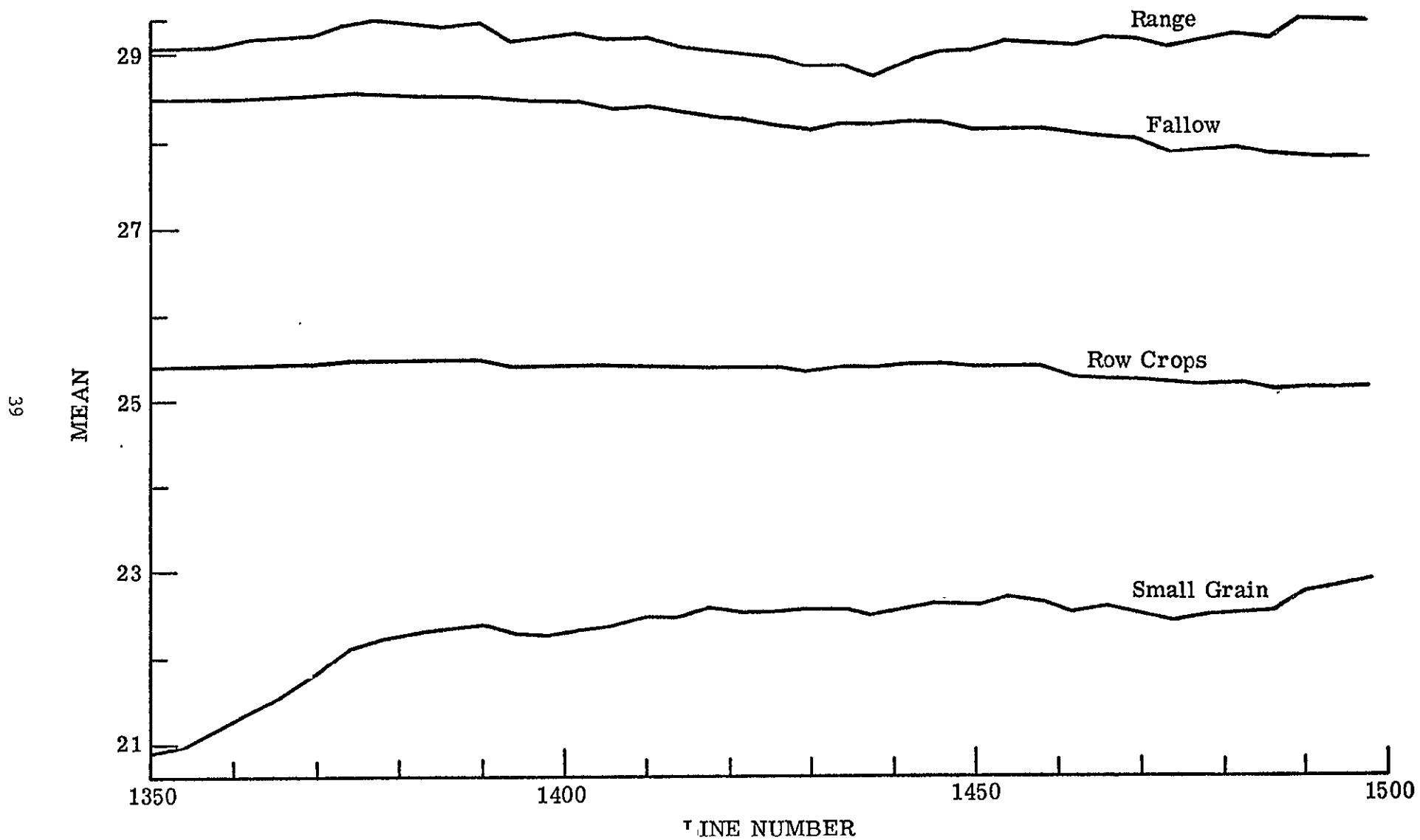
4.3 PROCESSING TIME REQUIRED FOR ADAPTIVE PROCESSING

An important consideration in adaptive processing is the additional processing time that the adapting requires. In Table 1, we compare processing times and accuracies for various operating modes. The preprocessing that was used was our standard scan angle correction program named ACORN. There is an approximately 11% processing time increase when adapting is used, which would reduce to approximately 3% had a quadratic decision rule been used rather than a linear decision rule. There is also an increase in processing time when we adapt the angle correction. For this data set, it appears that we gain by adapting the means, but adapting the angle correction is not as good as using ACORN.

4.4 EFFECTIVENESS OF MODIFICATION FOR COLORED NOISE

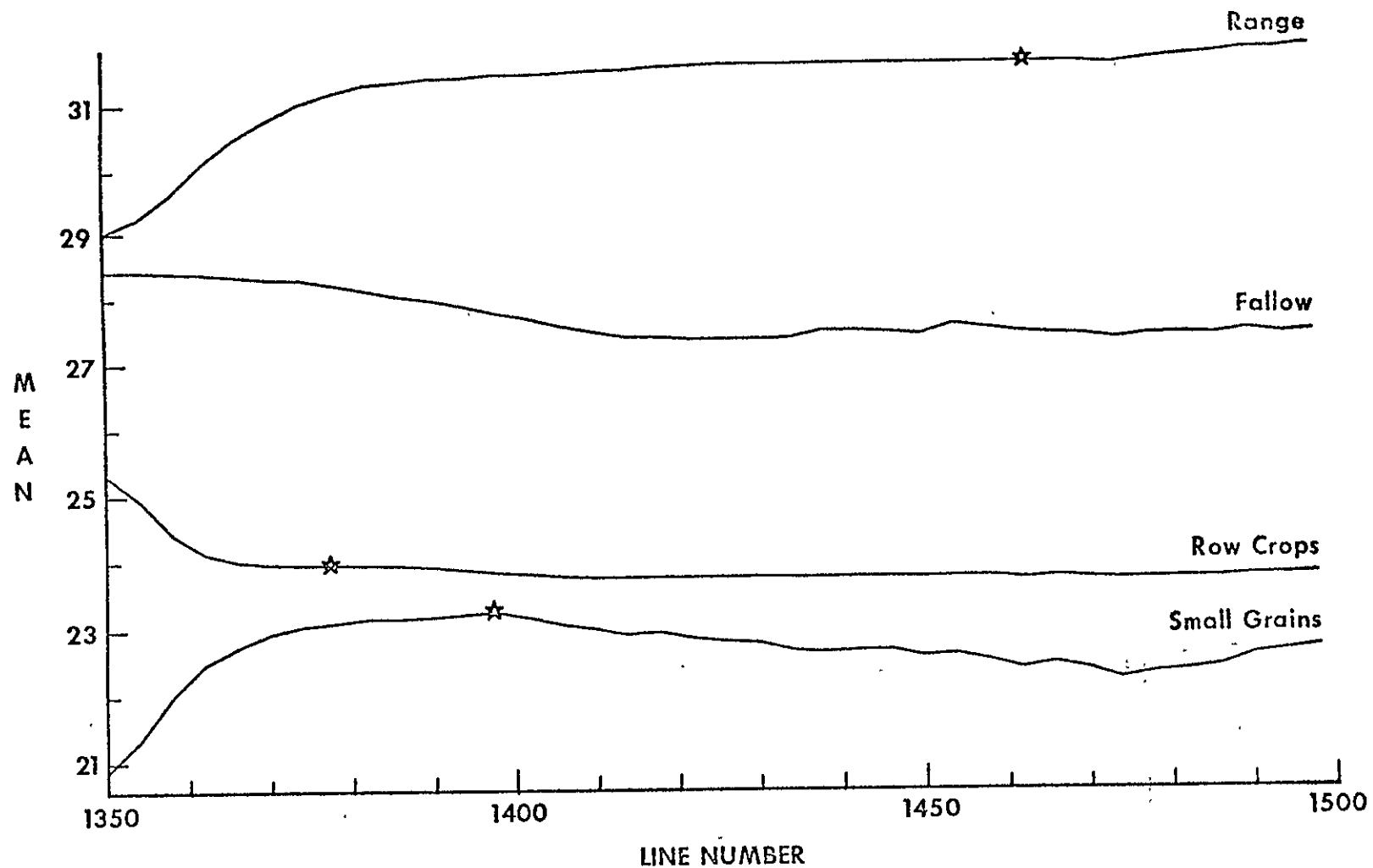
The algorithm considered here was described in detail in Section 3.4. It differs from the original adaptive processing algorithm principally in that it assumes that variations for different sampling times of the class mean vectors are correlated. A correlation length associated with this correlation is an input parameter to the program. Thus it may be set to approximate the average field length for the data being processed.

Programming of this algorithm was completed and tests were made to confirm the correctness of the program and the characteristics of the algorithm.



NORTH DAKOTA ERTS DATA CHANNEL 2 MEANS

FIGURE 9: VARIATIONS OF UPDATED MEANS WITHOUT USE OF AUXILIARY FIELD



N. DAKOTA ERTS DATA CHANNEL 2 MEANS

FIGURE 10: VARIATIONS OF UPDATED MEANS WITH USE OF AUXILIARY FIELDS



ALTERNATIVE CLASSIFICATION METHODS

	No Adapting No Preprocessing	No Adapting Preprocessing	Adapting No Preprocessing	Adapting Preprocessing	Adapting with Scan Angle Correction No Preprocessing
% CORRECT RECOGNITION	83.4	86.9	85.7	90.6	86.1
EXECUTION TIME (min.)	7	11	8	12	13

Figures based on classification into 5 classes using 5 channel aircraft data

TABLE 1

For these tests ERTS data from N. Dakota were used. This data set has been used extensively in tests for the original Kalman filter program. Fields in this data set are generally larger than those in the CITARS data sets, although probably not as large as could be found in major wheat growing data sets.

Storage requirements for the correlated state variation program are greater than for the original program. The state vector is twice as long (40 elements) and the error matrix 4 times larger (5 x 5 elements). This has not caused any problems. Storage requirements of the new program are well within the limitations of the 7094 computer (32 K words). Total processing time for the test data set was virtually the same as with the original adaptive program.

Figure 11 shows classification accuracy vs. θ_1 for both the original algorithm and the correlated state variation version. These figures are based on classification of 35 fields into 5 classes. Also shown for comparison is the accuracy obtained with conventional non-adaptive linear rule classification.

It can be seen that the new algorithm actually attains a slightly higher accuracy than the original, but at a much smaller value of θ_1 . This difference is in agreement with the theory presented in Section 3.4 which predicts that θ_1 for the new algorithm should be smaller by a factor of $1-\alpha^2$ to obtain the same updating rate. For this case $1-\alpha^2 \approx .02$.

Figures 12 and 13 show the effect of changing the correlation length parameter. In both figures the lower plot shows the change in the mean vector for a particular class and channel as a function of line number. The upper plot shows the change in the corresponding element of ω in equation 55.

In Figure 12 the value of n_0 was 30. In Figure 13 n_0 is increased to 90. As expected, the mean changes more rapidly in Figure 13 due to the higher effective updating rate caused by the higher value of n_0 .

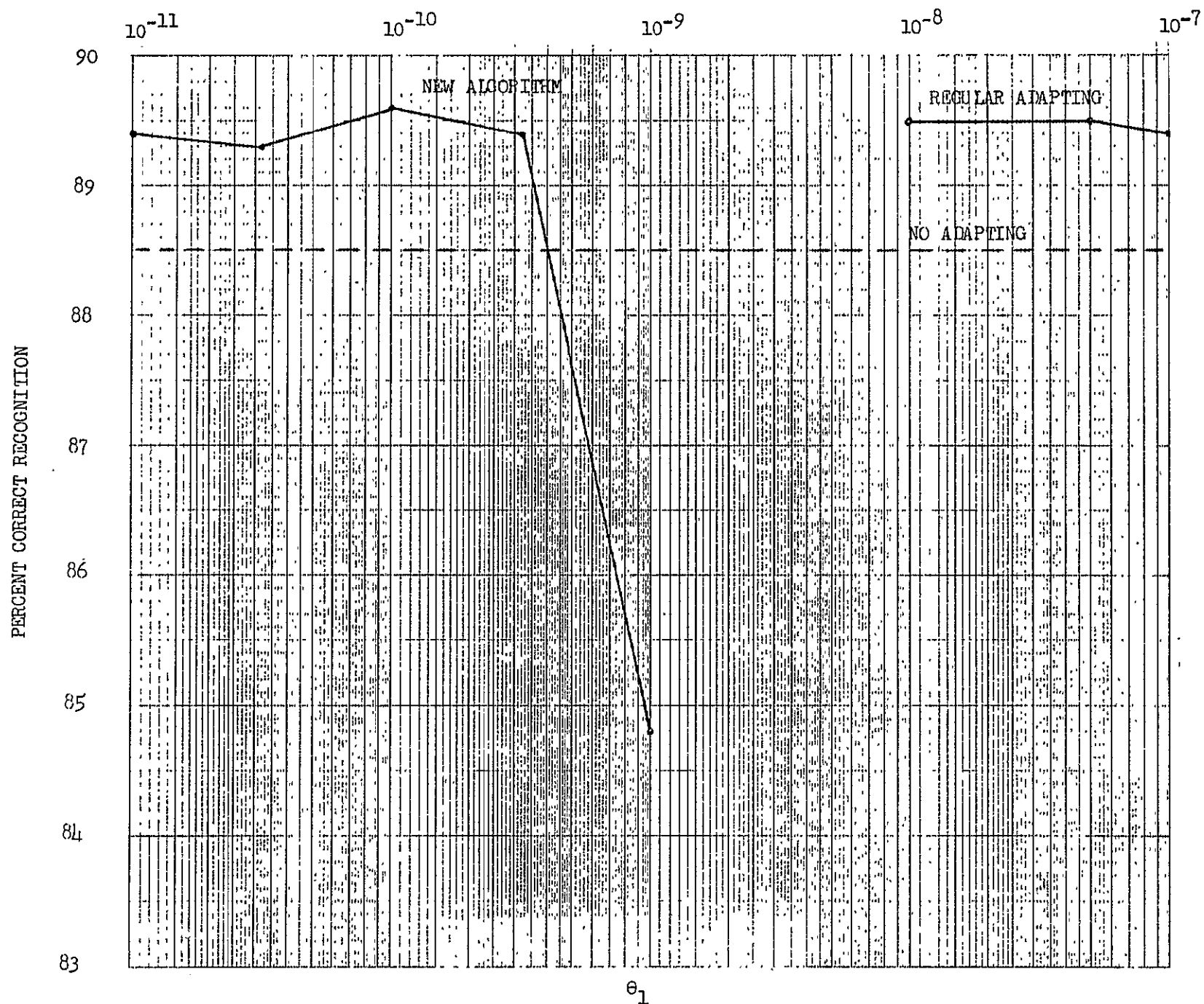


FIGURE 11. % CORRECT RECOGNITION vs. θ_1 FOR REGULAR AND CORRELATED STATE VARIATION ADAPTING

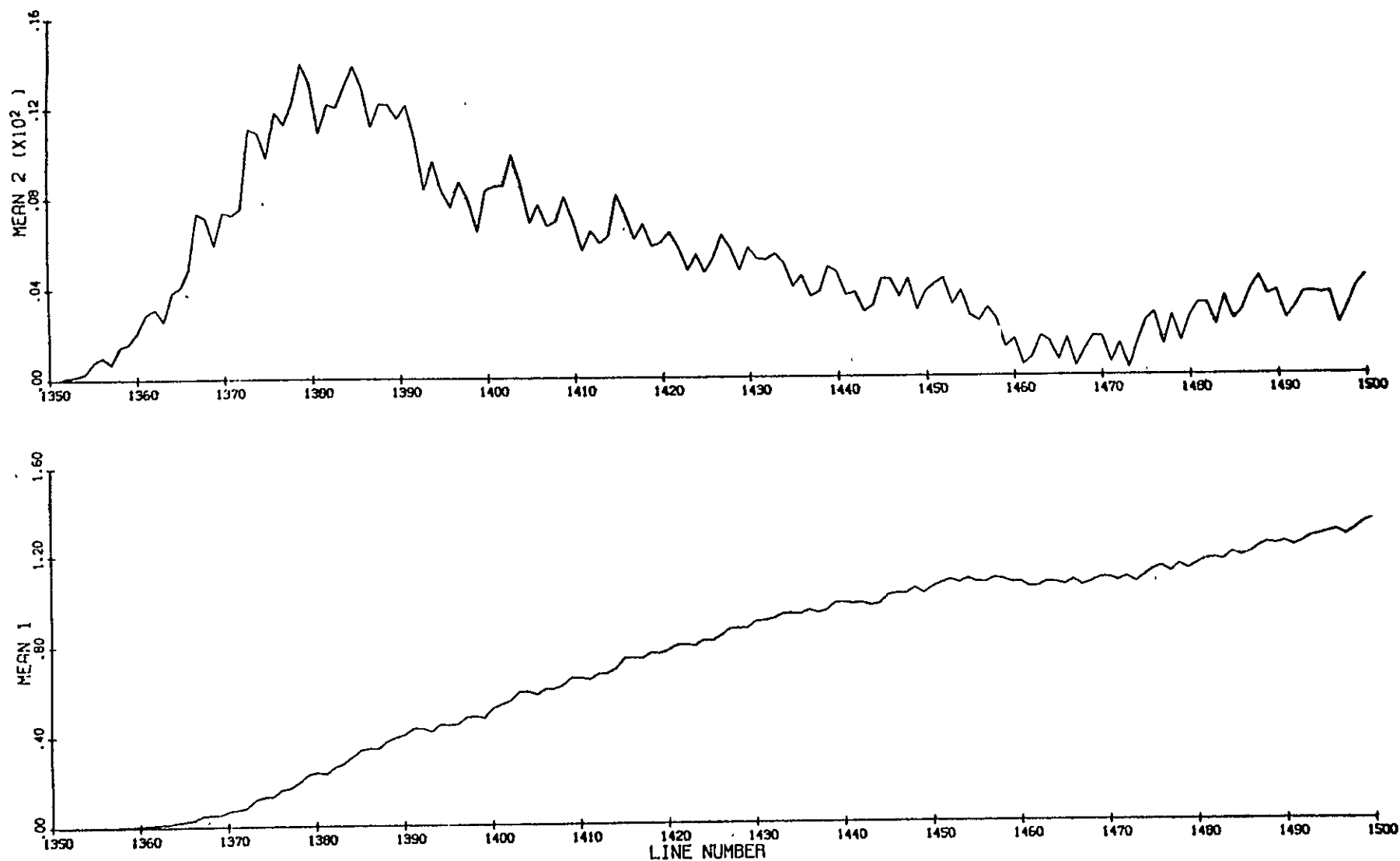


FIGURE 12. MEAN CHANGE VS.
LINE NUMBER
WITH $N_0 = 30$

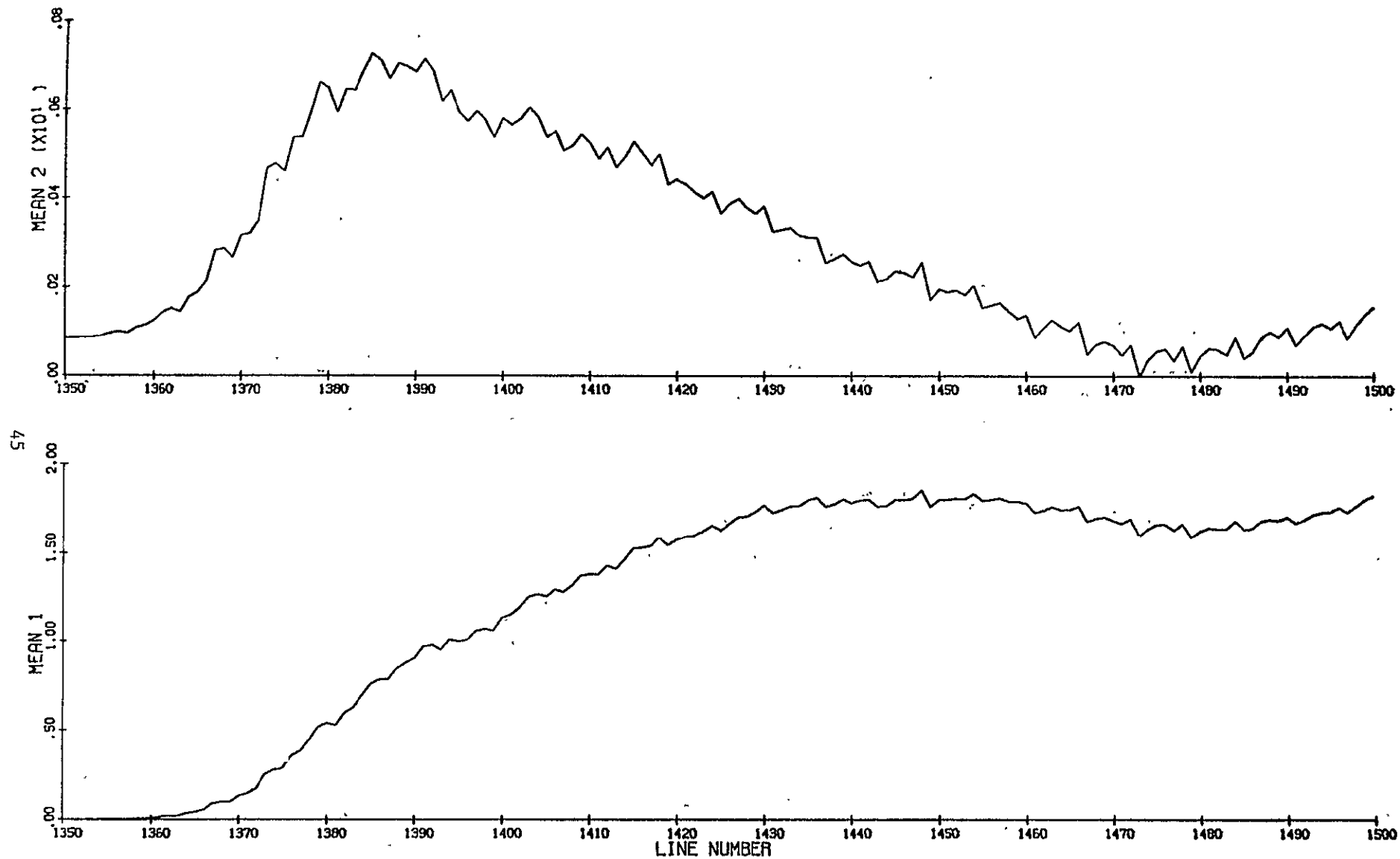


FIGURE 13. MEAN CHANGE VS.
LINE NUMBER
WITH $N_0 = 90$

4.5 ADAPTIVE PROCESSING LIMITATIONS

Further tests have been conducted to evaluate the performance of adaptive processing. For these tests both conventional linear rule classification and adaptive classification were performed on the same data set using signatures extracted from the data set being processed. Adaptive processing periodically updates the signature means on the basis of previous classification decisions to account for inter- and intra-field variations. Conventional classification uses the same means throughout the data set. To compare the recognition accuracy of the two methods, average percent correct recognition for a number of fields of known crop type was computed.

Previous comparisons of adaptive and conventional processing using LANDSAT data have shown that adaptive processing is capable of reducing classification errors by as much as one-third. The tests described below offer further confirmation of another previously reported result. That is, if two or more signatures are similar enough so that confusion exists between them with conventional classification, adaptive classifying will give even poorer results. However, if the signatures are reasonably distinct, adapting can generally be expected to improve classification accuracy.

Two LANDSAT data sets from the CITARS project were used for this testing. Fayette Co., Illinois data from 21 August and White Co., Indiana data from the same date were processed.

Table 2 shows the results of processing these data sets using both conventional and adaptive classification. The percentages shown are based on 125 fields from Fayette Co. and 166 fields from White Co. Signatures for six materials were obtained from the CITARS project.

TABLE 2. % CORRECT RECOGNITION WITH AND WITHOUT ADAPTING FOR FAYETTE AND WHITE CO. CITARS DATA

	<u>FAYETTE CO. NO ADAPTING</u>	<u>FAYETTE CO. ADAPTING</u>	<u>WHITE CO. NO ADAPTING</u>	<u>WHITE CO. ADAPTING</u>
CORN	87.4	89.5	72.0	71.2
SOY	85.5	85.5	77.7	77.7
OTHER*	68.7	66.8	90.4	90.4

*Other includes trees, bare soil, clover, and weeds.

The White Co. data is essentially unchanged by adapting. There is a slight (<1%) reduction in classification accuracy with adapting.

The Fayette Co. data show approximately 2% increase in accuracy of corn recognition with adapting and a corresponding 2% decrease in accuracy of the "other" class (trees, bare soil, clover and weeds).

An increase in recognition of one class and a corresponding decrease in another is characteristic of signature capture during adaptive processing. For these two data sets there is sufficient similarity in the signatures to cause capture. As has been noted previously, this precludes the successful use of the present adapting algorithm.

Capture can also occur with reasonably separated signatures if too rapid an updating rate is used. Figure 14 illustrates this. The solid line shows total recognition accuracy vs. θ_1 for Michigan LANDSAT data classified into 5 classes and representing 107 fields from Ionia and Clinton Co. The dashed line shows the percentage of corn correctly classified. Maximum accuracy is obtained with θ_1 in the range of 10^{-9} to 10^{-10} . Classification accuracy decreases for larger values of θ_1 (faster updating).

The dashed line shows a major cause of this decrease in accuracy. Virtually all of the corn pixels which were not classified as corn were classified as senescent vegetation, a class which included grass, field beans and alfalfa. The senescent vegetation signatures are close enough to the corn signature to cause just over 35% of the total corn pixels to be incorrectly classified as senescent vegetation even with small θ_1 . This causes the senescent vegetation means to be updated such that these signatures capture even more corn pixels. The higher the updating rate the more complete this capture process becomes. For $\theta_1 = 10^{-7}$ the senescent vegetation signatures have captured over 90% of the corn pixels.

Tables 3 and 4 show the classification results in greater detail for the two extremes of θ_1 . The left hand column gives the class name. This is followed by the number of fields and number of pixels actually belonging

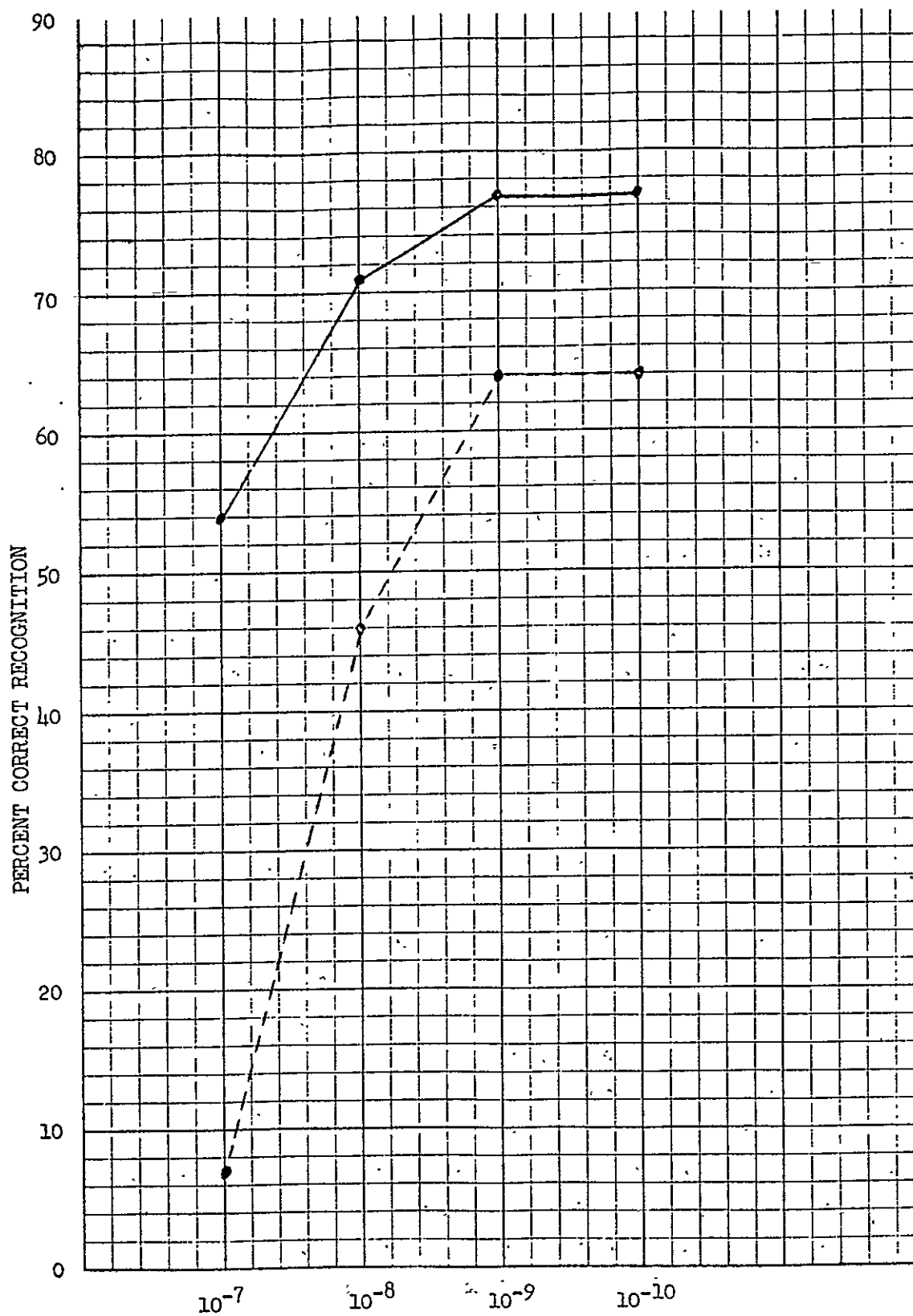


FIGURE 14. θ_1 % CORRECT RECOGNITION vs θ_1 FOR IONIA AND CLINTON CO. ERTS DATA

TABLE 3. CLASSIFICATION RESULTS WITH $\theta_1 = 10^{-7}$
 IONIA AND CLINTON CO. LANDSAT DATA

CLASS	NR. PLOTS	NR. POINT	SIGNATURES..			BARE SOIL	SENESC. VEG.
			CORN	SOY BEANS	TREES		
CORN	37	297	7.1			.3	92.6
SOYBEANS	6	27	3.7	66.7			29.6
TREES	6	47	38.3		19.1		42.6
BARE SOIL	10	53				100.0	
SENESC. VEG.	48	260		2.7		18.5	78.8
	107	684					

 TABLE 4. CLASSIFICATION RESULTS WITH $\theta_1 = 10^{-10}$
 IONIA AND CLINTON CO. LANDSAT DATA

CLASS	NR. PLOTS	NR. POINT	SIGNATURES..			BARE SOIL	SENESC. VEG.
			CORN	SOY BEANS	TREES		
CORN	37	297	64.0		.3		35.7
SOYBEANS	6	27	3.7	85.2			11.1
TREES	6	47	40.4	2.1	51.1		6.4
BARE SOIL	10	53				100.0	
SENESC. VEG.	48	260	.8	4.6		12.3	82.3
	107	684					

to that class. The remaining columns form a matrix showing what percentages of this total were assigned to the various classes.

Recall that the updating rate, as developed in [1] is defined as follows for updating after every point. Assume that a sufficient number of zero measurement vectors have been sampled so that the estimate of the first mean is the zero vector. Now, if the measurements become vectors composed entirely of the numbers, 1, the updating rate is the number of updates required for the mean estimate to become $1 - e^{-1}$. This number of updates has been shown to be approximately $\frac{1}{\sqrt{\theta_1}}$. A more useful form is

to express this in terms of lines rather than updates. The present program updates after every $1/3$ scan line, each of which contains N points. If we let the variance of each state variable be $N\theta_1$ because the state variation is assumed to occur after each line rather than after each update, then the updating rate becomes $\frac{1}{3\sqrt{MN\theta_1}}$ lines, for M materials. Updating rates on

the order of tens to hundreds of lines proved to be suitable for LANDSAT agricultural data.

Table 5 shows updating rate information for data sets from 4 different LANDSAT frames representing different geographical locations. In each case the value of θ_1 shown is the value which resulted in the maximum percent correct recognition for all known fields in the processed area. Parameters not shown were held constant while processing all 4 data sets.

For the North Dakota and Kansas data the updating rate is of the same order as the size of typical fields found in the scene. The Michigan and Illinois data required slower updating to avoid capture since the materials being classified were more easily confused than for the other data sets.

TABLE 5
OPTIMUM UPDATING RATES FOR 4 DATA SETS

<u>LOCATION</u>	<u>NUMBER OF MATERIALS</u>	<u>NUMBER OF POINTS PER LINE</u>	<u>θ_1</u>	<u>UPDATING RATE (LINES)</u>
North Dakota	5	650	5×10^{-8}	26
Kansas	3	101	10^{-5}	6
Michigan	7	310	10^{-8}	72
Illinois	6	126	10^{-9}	383

There is another parameter, ϕ_0 , that can have an effect on the classification accuracy, ϕ_0 is the initial value of the state error matrix in simplified form. It represents an estimate of the starting error in the material means. In normal use, the starting means are obtained from fields near the beginning of the area to be processed. In this case ϕ_0 may be assumed to be zero. However, if it is known that the starting means are not representative of the area being classified, then a non-zero ϕ_0 is appropriate. This situation might arise when means from one area are used to classify another geographically distant area. Thus ϕ_0 can be used to introduce a transient updating rate that is different from the steady-state rate.

Table 6 illustrates the use of non-zero values of ϕ_0 when the means being used are not representative. Four classification runs were made on a data set from the North Dakota LANDSAT frame. For the first two runs, signatures were used which were obtained from the data set being classified. The runs differed only in having different values of ϕ_0 . For the second two runs the signature means were increased by 5% to make them intentionally non-representative. Again two different values of ϕ_0 were used.

Table 6 shows percent correct recognition for the four runs. It can be seen that a zero value of ϕ_0 is preferable when the correct means are used but a non-zero ϕ_0 is preferable when the means are not correct.

TABLE 6. EFFECT OF TRANSIENT UPDATING RATE

<u>MEAN CHANGE</u>	<u>$\phi_o = 0$</u>	<u>$\phi_o = 10^{-2}$</u>
0.	92.7	90.4
+5	88.1	92.9

Another test of adaptive vs. conventional processing with CITARS data involved the use of signatures for Fayette Co., data of 11 June to classify Fayette Co. data of 10 June. The 11 June signatures were adjusted using a MASC (Multiplicative and Additive Signature Correction) transformation before being used to classify the 10 June data. The MASC transformation technique is explained in detail in [2]. The details of the process are not important to this test, however, since we are using the same signatures with both classifying techniques. What is significant is the choice of materials in this particular signature set. Signature for wheat, water, trees, bare soil, and weeds were included. These signatures are not as easily confused as the ones used in the test described above, so better results would be expected using adapting.

Table 7 shows the percentage of pixels correctly classified by conventional processing and adaptive processing for four classes. Three sets of figures are given for adaptive processing corresponding to different updating rates. Adaptive processing with $\theta_1 = 10^{-8}$ or 10^{-9} gives results somewhat better than conventional processing.

TABLE 7. % CORRECT RECOGNITION WITH AND WITHOUT
ADAPTING FOR FAYETTE CO. CITARS DATA

	NO ADAPTING	ADAPTING		
		$\theta_1=10^{-7}$	$\theta_1=10^{-8}$	$\theta_1=10^{-9}$
WHEAT	93	86	92	93
WATER	95	97	97	97
TREES	71	66	69	72
OTHER	88	91	90	88
AVERAGE	86.8	85.0	87.0	87.5

In another test, we tried adaptive processing on a data set for which the classification accuracy had been poor. The main cause of the poor accuracy was the classification of 38% of the corn data points as another class, namely trees. We hoped that we could increase the classification accuracy by inserting into the filter a large correlation coefficient between the corn and trees means.

The data set was collected in the same general area as that used for the surface water test. The classification accuracy with normal processing was 62%. With accuracy this poor, we would not expect that adaptive processing would be useful. Indeed, the accuracy was reduced to 51%, with 70% of the corn data misclassified as trees. Figure 7 provides further confirmation of the result that when normal processing has poor accuracy, adaptive processing has poorer accuracy. It is also worth repeating the converse, that when normal processing has high accuracy, adaptive processing improves the accuracy.

5

CONCLUSIONS AND RECOMMENDATIONS

The Kalman filter approach to adaptive processing appears to be useful for classification of LANDSAT data. The approach fails when the data are difficult to classify, because the successful operation of the filter depends upon the correctness of the decisions. However, when the decisions tend to be correct, each data point is used to update the proper mean vector which improves the decision rule, thereby improving classification accuracy.

The use of auxiliary fields appears to be useful when processing entire LANDSAT frames. When small portions of frames are to be processed, e.g., LACIE, then there is a choice between using ground-truthed fields as auxiliary fields or for signature extension. Tests should be conducted to determine which use of the fields is preferable.

The increases in computer processing time and memory requirements that adaptive processing causes are probably not significant. There are also negligible penalties in processing time and memory when using the colored noise modification.

For the limited amount of testing that has been performed, all of the Kalman filter modifications appear to perform the function for which they were designed. Exactly which modification should be used for a specific task should be determined for that task by additional testing. The testing that would be most appropriate at this time would be for the LACIE project. Let us now consider how such a test program might be conducted.

LACIE data would be used to perform experiments designed to test the following hypothesis: given a data set on which conventional linear rule recognition processing gives reasonably good classification accuracy, a decision-directed Kalman filter adaptive classifying algorithm will provide more accurate classification. More specifically we would compare the performance of conventional and adaptive processing in three different applications.

First, local recognition: For this case one or more intensive study sites would be classified using signatures extracted from the same site. Second, non-local recognition: In this case, signatures from one site would be used to classify a second site. Third, signature extension: Here, a large area containing two intensive study sites would be processed. These area would be selected so that one study site is near the beginning of the data to be processed and the other is near the end. Signatures would be obtained from the first site and classification accuracy would be determined for the second site.

For each of these tests we must consider the method of evaluation to be used and the selection of training and test areas. The evaluation technique would be the same for all three tests. For both conventional and adaptive processing the total acreage of the materials classified as well as percent correct recognition for all individually identified fields would be found,

For the local recognition tests, site selection is not critical. Any of the intensive study sites could be used. Training fields should be selected from the beginning of the site since the adapting algorithm assumes that the initial error in the means is zero. The study site could be divided into training and test sections at some arbitrary point leaving, for example, the first third of the site for training and the rest for test. Within these areas all fields large enough to contain at least as many field center (pure) pixels as data channels would be identified.

Non-local recognition tests would be restricted to the use of two or more intensive study sites found on the same LANDSAT frame. Four such instances are listed below:

<u>FRAME</u>	<u>DATE</u>	<u>INTENSIVE STUDY SITES</u>
1457-16551	23 October	Finney & Morton
1635-16395	19 April	Saline & Ellis
1689-16382	12 June 1974	Saline & Ellis
1725-16371	18 July	Saline & Ellis

The most stringent requirements are presented by the signature extension test. Here we need two study sites on the same quarter frame (tape) so that processing can be carried on uninterrupted from one site to the next. No quarter frame exists which contains any two of the Kansas sites. The situation for the Texas sites is not known to us since we do not yet have the tapes.

If no suitable tape contains two intensive study sites it may be necessary to use one study site and one SRS area for this test. SRS areas are smaller and less accurately examined than the intensive study sites but there are more of them.

We should add one final thought. The adaptive processing techniques discussed in this report have been tested using a multi-class decision rule. The techniques should apply directly to the LACIE decision rule, which is one that uses multiple signatures to form a two-class ratio test. When the ratios are formed, all of the quadratic functions needed for a multi-class decision rule are computed. The only computer functions that would be required to make a multi-class decision, which would be required when adaptive processing is to be used, are additions and amplitude comparisons. Consequently, the advantages of adaptive processing could be realized with only a small increase in processing time, probably less than a five percent increase.

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